

Math 24
Winter 2010
Monday, February 8

(1.) TRUE or FALSE?

(a.) The rank of a matrix is equal to the number of its nonzero columns.

(b.) The product of two matrices always has rank equal to the lesser of the ranks of the two matrices.

(c.) The $m \times n$ zero matrix is the only $m \times n$ matrix having rank 0.

(d.) Elementary row operations preserve rank.

(e.) Elementary column operations do not necessarily preserve rank.

(f.) The rank of a matrix is equal to the maximum number of linearly independent rows in the matrix.

(g.) The inverse of a matrix can be computed exclusively by means of elementary row operations.

(h.) The rank of an $m \times n$ matrix is at most the smaller of m and n .

(i.) An $n \times n$ matrix having rank n is invertible.

(2.) For each matrix, find the rank, and compute the inverse (if it exists):

(a.)
$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 2 & 3 & -1 \end{pmatrix}$$

(b.)
$$\begin{pmatrix} 0 & -2 & 4 \\ 1 & 1 & -1 \\ 2 & 4 & -5 \end{pmatrix}$$

(3.) Let A be an $m \times n$ matrix with rank m . Prove that there exists an $n \times m$ matrix B such that $AB = I_m$. (Hint: Think about the linear transformation L_A .)