Math 24
Winter 2010
Monday, February 1

(1.) TRUE or FALSE?

(a.) Suppose that $\beta = \{x_1, x_2, \ldots, x_n\}$ and $\beta' = \{x'_1, x'_2, \ldots, x'_n\}$ are ordered bases for a vector space and Q is the change of coordinate matrix that changes β' coordinates into β coordinates. Then the j^{th} column of Q is $[x_j]_{\beta'}$.

(b.) Every change of coordinate matrix is invertible.

(c.) Let T be a linear operator on a finite-dimensional vector space V, let β and β' be ordered bases for V, and let Q be the change of coordinate matrix that changes β' coordinates into β coordinates. Then $[T]_\beta = Q[T]_{\beta'}Q^{-1}$.

(d.) The matrices $A, B \in M_{n\times n}(F)$ are called similar if $B = Q^tAQ$ for some $Q \in M_{n\times n}(F)$.

(e.) Let T be a linear operator on a finite-dimensional vector space V. Then for any ordered bases β and γ for V, $[T]_\beta$ is similar to $[T]_\gamma$.

(f.) Suppose that $\beta = \{x_1, x_2, \ldots, x_n\}$ and $\beta' = \{x'_1, x'_2, \ldots, x'_n\}$ are ordered bases for a vector space and Q is the change of coordinate matrix that changes β' coordinates into β coordinates. Then $Q = [I_V]_{\beta'}^{\beta}$.

(g.) Every invertible matrix is a change of coordinate matrix.

For the next problems you may use the following fact (which you can check by multiplying these matrices together): If $ad - bc \neq 0$, then

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

(2.) Let α be the standard ordered basis for $\mathbb{R}^2$, β be the ordered basis $\{(1,2), (2,-1)\}$, and γ be the ordered basis $\{(1,-1), (1,1)\}$. Write down the change of coordinate matrices for changing:

β coordinates into
α coordinates.

α coordinates into
β coordinates.

γ coordinates into
α coordinates.

α coordinates into
γ coordinates.

β coordinates into
γ coordinates.

γ coordinates into
β coordinates.

(3.) Here α, β, and γ are the same ordered bases for $\mathbb{R}^2$ as in problem (2).

(a.) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation $T(a, b) = (a + b, a - b)$. Write down the matrices $[T]_\alpha$, $[T]_\beta$, $[T]_\gamma$, and $[T]_\beta^\gamma$.

(b.) Let $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$. Write down the matrices $[L_A]_\alpha$, $[L_A]_\beta$, $[L_A]_\gamma$, and $[L_A]_\beta^\gamma$.

(4.) If $\beta = \{2x^2 - x,\ 3x^2 + 1,\ x^2\}$ and $\beta' = \{1,\ x,\ x^2\}$ are ordered bases for $P_2(\mathbb{R})$, find the change of coordinate matrix that changes β' coordinates into β coordinates.