Math 24
Winter 2010
Monday, January 25

(1.) TRUE or FALSE? If V and W are finite-dimensional vector spaces over the field F, with ordered bases β and γ respectively, and T and U are linear transformations from V to W, then:

(a.) For any scalar a, the function $aT + U$ is a linear transformation from V to W.

(b.) $[T]_\beta^\gamma = [U]_\beta^\gamma$ implies that $T = U$.

(c.) If $m = dim(V)$ and $n = dim(W)$ then $[T]_\beta^\gamma$ is an $m \times n$ matrix.

(d.) $[T + U]_\beta^\gamma = [T]_\beta^\gamma + [U]_\beta^\gamma$.

(e.) $\mathcal{L}(V, W)$ is a vector space.

(f.) $\mathcal{L}(V, W) = \mathcal{L}(W, V)$.

(2.) Let β and γ be the standard ordered bases for $\mathbb{R}^n$ and $\mathbb{R}^n$ respectively. For each linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ compute $[T]_\beta^\gamma$.

(a.) $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(a, b) = (2a - b, 3a + 4b, a)$.

(b.) $T : \mathbb{R}^n \to \mathbb{R}^n$ defined by $T(a_1, a_2 \ldots, a_n) = (a_n, a_{n-1} \ldots, a_1)$.

(c.) $T : \mathbb{R}^n \to \mathbb{R}$ defined by $T(a_1, a_2 \ldots, a_n) = a_1 + a_n$.

(3.) Let β and γ be the standard ordered bases for $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. If $T : \mathbb{R}^2 \to \mathbb{R}^3$ and

$$[T]_\beta^\gamma = \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 2 & 3 \end{pmatrix}$$

then find a basis for $R(T)$.

(4.) If β is the standard basis for $\mathbb{R}^2$, and $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation with $[T]_\beta^\beta = \begin{pmatrix} a & b \\ c & c \end{pmatrix}$, what is $T(x, y)$? (Hint: Write (x, y) as a linear combination of the standard basis vectors.)

(5.) If $\beta = \{f_1, f_2, f_3\}$ is the basis for $P_2(\mathbb{R})$ (arrived at through the method of Lagrange interpolation) with the property that

$f_1(-1) = 1, \quad f_1(0) = 0, \quad f_1(2) = 0,$
$f_2(-1) = 0, \quad f_2(0) = 1, \quad f_2(2) = 0,$
$f_3(-1) = 0, \quad f_3(0) = 0, \quad f_3(2) = 1,$

find $[x^2]_\beta$ (the coordinates of x^2 in the basis β). (You do not have to compute the coefficients of f_1, f_2 and f_3 to do this problem.)

In general, for $f \in P_2(\mathbb{R})$, what is $[f]_\beta$?