Math 24
Winter 2010
Wednesday, January 20

The following question was asked in class today.

> If you have a vector v in a vector space V, can you always find a vector w in V such that $v = w + w$?

I gave the following partial answer.

> Yes, if V is a vector space over any of the fields $\mathbb{R}$ (the real numbers), $\mathbb{Q}$ (the rational numbers), or $\mathbb{C}$ (the complex numbers). This is because all those fields contain the number $\frac{1}{2}$, and so we can write
>
> $$v = \frac{1}{2}v + \frac{1}{2}v.$$
>
> Not necessarily, if V is a vector space over the two-element field F_2. For example, if $V = (F_2)^2$, then elements of V are pairs (a, b) where a and b are in F_2. In F_2 we have only two elements, 0 and 1, and we have $0+0 = 0$ and $1+1 = 0$. Therefore, in $(F_2)^2$, for every vector $w = (a, b)$ we have
>
> $$w + w = (a, b) + (a, b) = (a + a,\ b + b) = (0, 0),$$
>
> so there is no way to write the vector $(1, 1)$ in the form $w + w$.

The highly optional challenge question is this.

> Give a complete characterization of when, for a vector space V over the field F, there is a vector $v \in V$ that cannot be written in the form $v = w + w$ for some $w \in V$.

To answer this question, obviously you have to think about different kinds of fields. However, you do not need to know anything about fields beyond what is in the appendix of our textbook.