Subspaces

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Properties of Scalar Multiplication

Theorem. *In any vector space V, the following statements are true:*

1. $0x = 0$ *for each* $x \in V$.

2. $(-a)x = -(ax) = a(-x)$ *for each* a *in* F *and each* x *in* V.

3. $a0 = 0$ *for each* $a \in F$.

Subspaces

- A subset W of a vector space V over a field F is called a **subspace** of V if W is a vector space over F with the operations of addition and scalar multiplication defined on V.

Theorem. *Let V be a vector space and W a subset of V. Then W is a subspace of V if and only if the following three conditions hold for the operations defined in V.*

1. $0 \in W$.

2. $x + y \in W$ *whenever* $x \in W$ *and* $y \in W$.

3. $cx \in W$ *whenever* $c \in F$ *and* $x \in W$.

Theorem. *Any intersection of subspaces of a vector space* V *is a subspace of* V.