Systems of Linear Equations

Computational Aspects – Gauss Elimination

Lecture 20

February 23. 2007

Equivalent Systems of Equations

Definition

Two systems of linear equations are called equivalent if they have the same solution set.

Equivalent Systems of Equations

Theorem

Let $Ax = b$ be a system of m linear equations in n unknowns, and let C be an invertible $m \times m$ matrix. Then the system $(CA)x = Cb$ is equivalent to $Ax = b$.

Corollary

Let $Ax = b$ be a system of m linear equations in n unknowns. If $(A^{'}|b^{'})$ is obtained from $(A|b)$ by a finite number of elementary row operations, then the system $A^{'}x = b^{'}$ is equivalent to the original system.

The Reduced Row Echelon Form

Definition

A matrix is said to be in **reduced row echelon form** if the following three conditions are satisfied:

1. Any rows containing a nonzero entry orecedes any row in which all the entries are zero (if any).
2. The first nonzero entry in each row is the only nonzero entry in its column.
3. The first nonzero entry in each row is 1 and it occurs in a column to the right of the first nonzero entry in the preceding row.

Theorem

Gaussian elimination transforms any matrix into its reduced row echelon form.