Systems of Equations

Theoretical Aspects

Lecture 19

February 22, 2007

Systems of Equations

Definition

- A system of equations can be rewritten as a matrix equation

$$Ax = b.$$

- A **solution** to the system of equations is an n-tuple

$$s = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{pmatrix} \in F^n$$

such that $As = b$.

Definition

- The set of solutions is called **the solution set** of the system.
- A system of equation is called **consistent** if it has at least one solution.
- Otherwise it is called **inconsistent**.

Definition

- A system $Ax = b$ of m linear equations in n unknowns is called **homogeneous** if $b = 0$.
- Otherwise the system is called nonhomogeneous.

Theorem

Let $Ax = 0$ be a homogeneous system of linear equations. Let K denoted the solutions set of $Ax = 0$. Then $K = N(L_A)$; Hence K is a subspace of F^n of dimension $n - rank(L_A) = n - rank(A)$.

Corollary

If $m < n$, the system $Ax = 0$ has a nonzero solution.

Theorem

Let K be the solution set of a system of linear equations $Ax = b$, and let $K_H$be the solution set of corresponding homogeneous system $Ax = 0$. Then for any solution s to $Ax = b$

$$K = \{s\} + K_H.$$

Theorem

Let $Ax = b$ be a system of n linear equations in n unknowns. If A is invertible, then the system has exactly one solution, namely $A^{-1}b$. Conversely, if the system has one solution, then A is invertible.

Theorem

Let $Ax = b$ be a system of linear equations. Then the system is consistent if and only if $rank(A) = rank(A|b)$.