The Inverse of a Matrix

Lecture 18

February 21, 2007

The Augmented Matrix

Definition

- Let A and B be $m \times n$ and $m \times p$ matrices, respectively.
- The **augmented matrix** $(A|B)$ is the $m \times (n + p)$ matrix $(A\ B)$.

The Inverse of a Matrix

Fact

- *If A is an invertible $n \times n$ matrix, then it is possible to transform the matrix $(A|I_n)$ into the matrix $(I_n|A^{-1})$ by means of a finite number of tow operations.*
- *If A is an invertible $n \times n$ matrix, and the matrix $(A|I_n)$ is transformed into a matrix of the form $(I_n|B)$ by means of a finite number of elementary row operations, then $B = A^{-1}$.*

Definition

- A system of equations can be rewritten as a matrix equation

$$Ax = b.$$

- A **solution** to the system of equations is an n-tuple

$$s = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{pmatrix} \in F^n$$

such that $As = b$.

Definition

- The set of solutions is called **the solution set** of the system.
- A system of equation is called **consistent** if it has at least one solution.
- Otherwise it is called **inconsistent**.

Definition

- A system $Ax = b$ of m linear equations in n unknowns is called **homogeneous** if $b = 0$.
- Otherwise the system is called nonhomogeneous.

Theorem

Let $Ax = 0$ be a homogeneous system of linear equations. Let K denoted the solutions set of $Ax = 0$. Then $K = N(L_A)$; Hence K is a subspace of F^n of dimension $n - rank(L_A) = n - rank(A)$.

Corollary

If $m < n$, the system $Ax = 0$ has a nonzero solution.