The Inverse of a Matrix

Lecture 18

February 21, 2007

The Augmented Matrix

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- The **augmented matrix** $(A|B)$ is the $m \times (n+p)$ matrix $(A\ B)$.

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- *If A is an invertible $n \times n$ matrix, and the matrix $(A|I_n)$ is transformed into a matrix of the form $(I_n|B)$ by means of a finite number of elementary row operations, then $B = A^{-1}$.*

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- A **solution** to the system of equations is an n-tuple

$$s = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{pmatrix} \in F^n$$

such that $As = b$.

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- Otherwise it is called **inconsistent**.

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- Otherwise the system is called nonhomogeneous.

Theorem

Let $Ax = 0$ be a homogeneous system of linear equations. Let K denoted the solutions set of $Ax = 0$. Then $K = N(L_A)$; Hence K is a subspace of F^n of dimension $n - rank(L_A) = n - rank(A)$.

Corollary

If $m < n$, the system $Ax = 0$ has a nonzero solution.