Math 24

Homework 9 (not to be turned in)

1. Let $u_1, \ldots, u_p$ be an orthogonal basis for a subspace W of $\mathbb{R}^n$ and let $T : \mathbb{R}^n \to \mathbb{R}^n$ be defined by $T(x) = \text{proj}_W\, x$. Show that T is a linear transformation.

2. Determine if $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ is an orthogonal matrix, and if so find its inverse.

3. Determine if $\begin{bmatrix} .5 & .5 & -.5 & -.5 \\ .5 & .5 & .5 & .5 \\ .5 & -.5 & -.5 & .5 \\ 5 & -.5 & .5 & -.5 \end{bmatrix}$ is an orthogonal matrix, and if so find its inverse.

4. Orthogonally diagonalize the matrix $\begin{bmatrix} 1 & -6 & 4 \\ -6 & 2 & -2 \\ 4 & -2 & -3 \end{bmatrix}$ giving an orthogonal matrix P and a diagonal matrix D. The eigenvalues are: $-3, -6, 9$.

5. Orthogonally diagonalize the matrix $\begin{bmatrix} 4 & 0 & 1 & 0 \\ 0 & 4 & 0 & 1 \\ 1 & 0 & 4 & 0 \\ 0 & 1 & 0 & 4 \end{bmatrix}$ giving an orthogonal matrix P and a diagonal matrix D. The only eigenvalues are 3, 5.

6. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Verify that $v_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ are eigenvectors of A. Then orthogonally diagonalize A.

7. Find the SVD of $\begin{bmatrix} 4 & 6 \\ 0 & 4 \end{bmatrix}$.

8. Find the SVD of $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$. Hint: One column of U can be $\begin{bmatrix} 1/\sqrt{6} \\ -2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$.

9. Suppose the factorization below is an SVD of a matrix A, with the entries rounded to two places.

$$A = \begin{bmatrix} .40 & -.78 & .47 \\ .37 & -.33 & -.87 \\ -.84 & -.52 & -.16 \end{bmatrix} \begin{bmatrix} 7.10 & 0 & 0 \\ 0 & 3.10 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} .30 & -.51 & -.81 \\ .76 & .64 & -.12 \\ .58 & -.58 & .58 \end{bmatrix}.$$

   (a) What is the rank of A?

(b) Use this decomposition of A with no calculation to write a basis for $\operatorname{Col} A$ and a basis for $\ker A$.

10. Repeat the above exercise for the SVD of the following 3×3 matrix A:

$$A = \begin{bmatrix} -.86 & -.11 & -.50 \\ .31 & .68 & -.67 \\ .41 & -.73 & -.55 \end{bmatrix} \begin{bmatrix} 12.48 & 0 & 0 & 0 \\ 0 & 6.34 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} .66 & -.03 & -.35 & .66 \\ -.13 & -.90 & -.39 & -.13 \\ .65 & .08 & -.16 & -.73 \\ -.34 & .42 & -.84 & -.08 \end{bmatrix}.$$

11. Suppose that A is square and invertible. Find a singular value decomposition of A^{-1}.