

Math 24
Homework 7

#6.3.2c Find the characteristic polynomial and all eigenvalues and eigenvectors for $A = \begin{bmatrix} 2 & 1 & 1 \\ -4 & -3 & 0 \\ -2 & -2 & 1 \end{bmatrix}$.

#6.3.4 [(a) Prove that the matrices $\begin{bmatrix} 1 & -5387621.4 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 5 & 3 \\ -4 & -2 \end{bmatrix}$ are similar.

[(b)] Prove that the matrices $\begin{bmatrix} 1.000000001 & 0 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ are not similar.

#4.1.1c,d Use Theorem 4.3 to show the following sets are linearly independent:

(c) $\left\{ \begin{bmatrix} 2 & 0 & 1 \\ 0 & -3 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 2 & 6 \\ -1 & 2 & 0 \end{bmatrix}, \begin{bmatrix} -2 & 3 & 0 \\ 6 & 0 & 1 \end{bmatrix} \right\} \subset M_{2 \times 3}(\mathbb{R})$.

(d) $\{5x^2 - 6x + 1, x^2 - 2x + 1, 10x^2 - 8x + 1\} \subset C([0, 1])$.

#4.1.12 Let V be an inner product space.

(a) Show that $v = 0$ if and only if $\langle v, w \rangle = 0$ for every $w \in V$.

(b) Show that $v = w$ if and only if $\langle v, u \rangle = \langle w, u \rangle$ for every $u \in V$.

(c) Let $S, T \in \mathcal{L}(V)$. Show that $S = T$ if and only if $\langle S(v_1), v_2 \rangle = \langle T(v_1), v_2 \rangle$ for every $v_1, v_2 \in V$.

#4.2.4ab Find the coordinate representation of each of the following vectors with respect to the orthonormal basis given in the corresponding part of Exercise 4.2.2.

(a) $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 7 \\ 7 & 2 \end{bmatrix}$

#4.2.6ab Find the matrix representing each of the following linear maps with respect to the orthonormal basis given in the corresponding part of Exercise 4.2.2.

(a) $T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y \\ z \\ x \end{bmatrix}$ (b) $T(A) = A^T$.