Your Name Goes Here

Math 24

Homework 6

#3.5.12 (a) Show that $\mathcal{B} = \{1, x, \frac{3}{2}x^2 - \frac{1}{2}\}$ is a basis for $\mathcal{P}_2(\mathbb{R})$.

(b) Find the coordinate representation of x^2 with respect to $\mathcal{B}$.

(c) Let $D : \mathcal{P}_2(\mathbb{R}) \to \mathcal{P}_2(\mathbb{R})$ be the derivative operator. Find the coordinate representation of D with respect to $\mathcal{B}$.

(d) Use your answers to the last two parts to compute $\frac{d}{dx}(x^2)$.

#3.6.4 Consider the basis $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$ of $\mathbb{R}^2$.

(a) Find the change of basis matrices $[I]_{\mathcal{B},\mathcal{E}}$ and $[I]_{\mathcal{E},\mathcal{B}}$.

(b) Use these change of basis matrices to find the coordinate representation of each of the following vectors with respect to $\mathcal{B}$.

$$(i) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (ii) \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (iii) \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad (iv) \begin{bmatrix} 4 \\ -5 \end{bmatrix} \quad (v) \begin{bmatrix} -6 \\ 1 \end{bmatrix}$$

#8a,b(iii) Consider the basis $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}$ of $\mathbb{R}^2$, and $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ of $\mathbb{R}^3$, and the linear maps $S \in \mathcal{L}(\mathbb{R}^2, \mathbb{R}^3)$ and $T \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$ given (wrt standard bases) by

$$[S]_{\mathcal{E},\mathcal{E}} = \begin{bmatrix} 2 & -1 \\ 5 & 3 \\ -3 & 2 \end{bmatrix} \qquad \text{and} \qquad [T]_{\mathcal{E},\mathcal{E}} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}.$$

(a) Find the change of basis matrices $[I]_{\mathcal{B},\mathcal{E}}$ and $[I]_{\mathcal{E},\mathcal{B}}$ for $\mathbb{R}^2$, and $[I]_{\mathcal{C},\mathcal{E}}$ and $[I]_{\mathcal{E},\mathcal{C}}$ for $\mathbb{R}^3$.

(8biii) Use these change of basis matrices to find the coordinate representation of (iii) $[S]_{\mathcal{B},\mathcal{C}}$.

#3.6.10 Consider the bases $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} \right\}$ of the subspace U of $\mathbb{R}^3$ defined by $x + 2y + 3z = 0$, and let $T \in \mathcal{L}(U)$ be given by $T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9z \\ 3y \\ x \end{bmatrix}$.

(a) Find the matrix $[T]_{\mathcal{B}}$.

(b) Find the change of basis matrices $[I]_{\mathcal{B},\mathcal{C}}$ and $[I]_{\mathcal{C},\mathcal{B}}$.

(c) Use your answers to the previous parts to find $[T]_{\mathcal{C}}$.

#3.6.14 Consider the bases $\mathcal{B} = \{v_1, \ldots, v_n\}$ and $\mathcal{C} = \{w_1, \ldots, w_n\}$ of $\mathbb{F}^n$ from the example on page 201.

(a) Find the change of basis matrix $[I]_{\mathcal{C},\mathcal{B}}$.

#3.6.16 Prove that the matrices

$$\begin{bmatrix} 2 & 0 & -1 \\ -1 & 3 & 2 \\ 1 & 3 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & 0 & -1 \\ 2 & -3 & -1 \\ 1 & 3 & 1 \end{bmatrix}$$

are not similar to each other.

#3.6.18 Show that the matrix $\begin{bmatrix} 5 & 6 \\ -1 & -2 \end{bmatrix}$ is diagonalizable.

#6.1.8 Let $R : \mathbb{R}^3 \to \mathbb{R}^3$ denote reflection across the plane $x + 2y + 3z = 0$. What is $\det R$?

#6.1.9 Suppose that $\dim V = n$ and $T \in \mathcal{L}(V)$ has n distinct eigenvalues $\lambda_1, \ldots, \lambda_n$. Prove that $\det T = \lambda_1 \cdots \lambda_n$.

#6.2.2c Find the determinant of $\begin{bmatrix} 1 & -2 & 3 & 0 \\ -2 & 1 & 0 & -1 \\ 3 & -4 & 5 & -2 \\ 0 & -1 & 2 & -3 \end{bmatrix}$.