Your Name Goes Here

Math 24

Homework 4

#3.1.9 Suppose that $T \in \mathcal{L}(V, W)$ is injective and that $\{v_1, \ldots, v_n\}$ is linearly independent in V. Show that $\{T(v_1), \ldots, T(v_n)\}$ is linearly independent in W.

#3.1.12 Suppose that $A \in M_n(\mathbb{F})$ is upper triangular (see Exercise 2.3.12) and that all the diagonal entries are nonzero (i.e., $a_{ii} \neq 0$ for all $i = 1, \ldots, n$.) Use Corollary 3.7 to prove that the columns of A are linearly independent.

#3.1.14 Let $n \geq 1$ be an integer, and suppose that there are constants $a_1, \ldots, a_n \in \mathbb{R}$ such that

$$\sum_{k=1}^{n} a_k \sin(kx) = 0$$

for every $x \in \mathbb{R}$. Show that $a_1 = \cdots = a_n = 0$.
Hint: Consider the linear map $D^2 : C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R})$ given by $D^2(f) = f''$, where f'' is the second derivative and $C^\infty(\mathbb{R})$ is the space of infinitely differentiable functions on $\mathbb{R}$, and use Theorem 3.8.

#3.2.10 Find the unique matrix $A \in M_2(\mathbb{R})$ such that $A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ and $A \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$.

#3.2.16 Find a sublist of $\{x + 1, x^2 - 1, x^2 + 2x + 1, x^2 - x\}$ which is a basis for $\mathcal{P}_2(\mathbb{R})$.

#3.2.22 Suppose that $\{v_1, \ldots, v_n\}$ and $\{w_1, \ldots, w_n\}$ are both bases for V. Show that there is an isomorphism $T \in \mathcal{L}(V)$ so that $T(v_i) = w_i$ for all i.