Your Name Goes Here

Math 24

Homework 2

#2.1.5 Let $C(\mathbb{R})$ be the vector space over $\mathbb{R}$ of continuous functions $f : \mathbb{R} \to \mathbb{R}$. Define $T : C(\mathbb{R}) \to C(\mathbb{R})$ by $[Tf](x) = f(x)\cos x$. Show that T is a linear map.

#2.1.7 Verify that $\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{bmatrix}$, and determine the corresponding eigenvalue.

#2.1.9c Find all the eigenvalues and eigenvectors (over $\mathbb{C}$) of the matrix $\begin{bmatrix} 2 & -7 \\ 0 & 2 \end{bmatrix}$.

#2.1.11 Find all eigenvalues and eigenvectors of the orthogonal projection $P \in \mathcal{L}(\mathbb{R}^2)$ described in Exercise 2.1.2.

#2.1.12 ($n = 2, 3$ only) Find all the eigenvalues and eigenvectors of the matrix $A \in M_n(\mathbb{R})$ with $a_{ij} = 1$ for all i, j, **and** $n = 2, 3$.

#2.1.13 Consider a linear system in matrix form: $A\mathbf{x} = \mathbf{b}$ where $A \in M_{m\times n}(\mathbb{F})$ and $\mathbf{b} \in \mathbb{F}^m$ are given. Suppose that $\mathbf{x}, \mathbf{y} \in \mathbb{F}^m$ are both solutions. Under what condition is $\mathbf{x} + \mathbf{y}$ also a solution?

#2.1.14 Suppose that $A \in M_n(\mathbb{F})$ and that for each $j = 1, \dots, n$, $\mathbf{e}_j$ is an eigenvector of A. Prove that A is a diagonal matrix.

#2.2.2 Give an explicit isomorphism between $\mathbb{R}^2$ and the set of solutions of the linear system (over $\mathbb{R}$)

$$\begin{aligned} w - x + 3z &= 0 \\ w - x + y + 5z &= 0 \\ 2w - 2x - y + 4z &= 0. \end{aligned}$$

#2.2.7 Find the matrix of the linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$ which first reflects across the y-axis, then rotates counterclockwise $\pi/4$ radians, then stretches by a factor of 2 in the y direction.

#2.2.8 Show that if 0 is an eigenvalue of $T \in \mathcal{L}(V)$, then T is not injective.