Quiz 7

MATH 24 — SPRING 2014

Sample Solutions

Let $A = \begin{pmatrix} 1+i & 1 \\ 0 & 1-i \end{pmatrix}$.

1.– Compute an eigenvector for the eigenvalue $1+i$.

Solution. The null space of $A-(1+i)I = \begin{pmatrix} 0 & 1 \\ 0 & -2i \end{pmatrix}$ is $\operatorname{span}\{e_1\}$. □

2.– Compute an eigenvector for the eigenvalue $1-i$.

Solution. The null space of $A-(1+i)I = \begin{pmatrix} 2i & 1 \\ 0 & 0 \end{pmatrix}$ is $\operatorname{span}\{(-i/2)e_1 + e_2\}$. □

3.– Compute the (standard) inner product of the two eigenvectors you just found.

Solution. $\langle e_1, (-i/2)e_1 + e_2\rangle = \langle e_1, (-i/2)e_1\rangle + \langle e_1, e_2\rangle = (i/2)\langle e_1, e_1\rangle + \langle e_1, e_2\rangle = i/2.$ □

4.– Explain why A is not normal.

Solution. If A were normal, its eigenspaces would be orthogonal to each other by Theorem 6.15(d). Part 3 shows that this is not the case, so A cannot be normal. Note that A is still diagonalizable since it has two distinct eigenvalues. □