Math 24
Spring 2012

Quiz Sample Solutions

Monday, May 21

1. In the inner product space $\mathbb{C}^2$ with the standard inner product, compute $||(1,i)||$.

$$||(1,i)|| = \sqrt{\langle (1,i),\, (1,i)\rangle} = \sqrt{(1)(1)+(i)(-i)} = \boxed{\sqrt{2}}$$

2. Suppose $\beta = \{v_1,\, v_2,\, v_3\}$ is an orthonormal basis for $\mathbb{R}^3$, and $v_1 = \left(\frac{3}{13},\, \frac{4}{13},\, \frac{12}{13}\right)$. If the coordinates of $(1,1,1)$ in basis β are $[(1,1,1)]_\beta = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$, what is a?

$$a = \left\langle (1,1,1),\, \left(\frac{3}{13}, \frac{4}{13}, \frac{12}{13}\right)\right\rangle = \boxed{\frac{19}{13}}$$

3. Suppose the vector space $P_1(\mathbb{R})$ is given the inner product

$$\langle p(x), q(x)\rangle = \int_{-1}^{1} p(x)q(x)\,dx.$$

If $W = span(1)$, find a basis for $W^\perp$.

$$\langle 1, x\rangle = \int_{-1}^{1} x\,dx = 0 \text{ so a basis for } W^\perp \text{ is } \boxed{\{x\}}$$

4. If $A \in M_{n\times n}(\mathbb{R})$, which of the following are equivalent to "A is diagonalizable"? Circle all correct answers.

   (a) The characteristic polynomial of A splits.

   (b) $\boxed{\mathbb{R}^n \text{ is the direct sum of eigenspaces of } A.}$

   (c) $\boxed{\mathbb{R}^n \text{ has a basis consisting of eigenvectors of } A.}$