Math 24
Spring 2012
Quiz
Monday, April 23

Sample Solutions

1. If $A = \begin{pmatrix} 1 & 0 \\ 2 & -2 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 5 & 7 \\ -1 & 3 & 0 \end{pmatrix}$, find the following entries in the product matrices.

   $(AB)_{12} = \boxed{5}$ $\qquad$ $(BA)_{12} = \boxed{-3}$

2. Suppose $T : V \to W$ is a linear function between n-dimensional vector spaces over a field F. List three different conditions, other than "T is one-to-one" and "T is onto," that will guarantee T is invertible.

   $n(T) = 0$

   $r(T) = n$

   $N(T) = \{0\}$

   T takes linearly independent sets to linearly independent sets

   T takes a basis for V to a basis for W

   $[T]_\alpha^\beta$ is invertible (where α and β are bases for V and W)

   $L_{[T]_\alpha^\beta}$ is invertible

   . . . and many other possibilities

3. If $A \in M_{2\times 3}(\mathbb{C})$, then

   The domain of L_A is $\boxed{\mathbb{C}^3}$ $\qquad$ The range of L_A is $\boxed{\mathbb{C}^2}$

4. TRUE OR $\boxed{FALSE}$: If $\beta = \{(1,1,0), (1,0,1), (0,1,1)\}$ is a basis for $\mathbb{Q}^3$, then $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ is the matrix that changes from standard coordinates to β-coordinates.