1. (10) Let V and W be vector spaces over a field F, and let $T : V \to W$ be a linear transformation. Give each of the terms below a precise mathematical definition.

   (a) Define what is meant by the **range of** T.

   (b) Define what is meant by saying that S is an **inverse (function)** to T. Caution: do not characterize conditions under which S exists, but define what it means to be an inverse function.

2. (15) Let V and W be finite-dimensional vector spaces over a field F, and let $T : V \to W$ be a linear transformation. Suppose that $\dim V > \dim W$. Characterize each of the following statements by using one of the following terms: **always**, **never**, or **sometimes**. For each answer of **always** or **never**, give a brief argument justifying your response; for each answer of **sometimes**, give two examples—one showing where the condition holds, and one showing where it does not.

   (a) T is one-to-one (injective)

   (b) T is onto (surjective)

3. (25) (Short Answer)

(a) Let $V = P_2(\mathbb{R})$, and let $T : V \to V$ be the linear map defined by $T(f) = f + f'$ (f' is the first derivative). Find the matrix of T with respect to the ordered basis $\{1, x, x^2\}$ of V.

(b) Suppose that V and W are vector spaces over a field F, with ordered bases $\mathcal{B}_V = \{v_1, v_2\}$ and $B_W = \{w_1, w_2\}$ respectively. Let $T : V \to W$ be a linear transformation such that $[T]_{\mathcal{B}_V}^{\mathcal{B}_W} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. If $v = 3v_1 - 2v_2$, express $T(v)$ as a linear combination of w_1 and w_2.

(c) Suppose that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation, and that with respect to the standard basis $\mathcal{B}$, T has matrix $[T]_{\mathcal{B}} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. Find $T(1, 0, 3)$.

(continued on next page)

(d) Suppose that the matrix $A \in M_{3\times 4}(\mathbb{R})$ has row-reduced echelon form $\begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

Is the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ in $R(L_A)$? (Briefly justify your answer)

(e) Let V, W and Z be vector spaces over a field F, and let $T : V \to W$ and $S : W \to Z$ be linear maps. Suppose that ST is one-to-one. You showed for homework that T must be one-to-one. Show by example that S need not be one-to-one.

Math 24

22 February 2001
Second Hour Exam
(In class part)

NAME (Print!): ______________________________

Problem	Points	Score
1	10	
2	15	
3	25	
4	10	
5	10	
6	10	
7	10	
8	10	
Total	100	