1. (10) Let V be a finite dimensional vector space and S a nonempty (but not necessarily finite) subset of V.

   (a) Define what it means for the set S to be linearly dependent.

   (b) Define what is meant by a basis for V. (Be sure to define any mathematical terms you use in your definition which originate in this course).

2. (40) Modified True/False. Circle the correct response (**T**rue or **F**alse). Then, if true, give a brief explanation; if false, give a counterexample. Remember, true means true in all cases.

T **F** Let S and T be nonempty subsets of a vector space V. If T is a linearly dependent subset of V and $S \subseteq T$ then S is linearly dependent.

T **F** If V is an n-dimensional vector space, then every set of $n+1$ **nonzero** vectors in V spans V.

T **F** Let V be a finite-dimensional vector space, W a subspace of V, and $\mathcal{B} = \{v_1, \ldots, v_n\}$ a basis for V. Then there is a subset $S \subseteq \mathcal{B}$ whose span is W.

(continued on next page)

(Problem 2 continued)

T **F** Let V be a finite-dimensional vector space of dimension n. Let $S = \{v_1, \ldots, v_n\}$ and $T = \{w_1, \ldots, w_n\}$ be two linearly independent subsets of V. Then $\text{Span}(S) = \text{Span}(T)$.

T **F** If U and W are distinct subspaces (i.e., $U \neq W$) of a finite-dimensional vector space V such that $\dim(U) + \dim(W) = \dim(V) = n$, then there is a basis $\{u_1, \ldots, u_k\}$ for U and a basis $\{w_1, \ldots, w_{n-k}\}$ for W, so that $\{u_1, \ldots, u_k, w_1, \ldots, w_{n-k}\}$ is a basis for V.

T **F** Let V be a 5-dimensional vector space. Let W_1 and W_2 be subspaces of dimension 3 and 4 respectively. Then $W_1 \cap W_2 \neq \{0\}$.

(continued on next page)

(Problem 2 continued)

T **F** Let V be a vector space of (finite) dimension n. There exist subspaces of V of dimensions 0, 1, 2, ..., n.

T **F** Let A be a nonzero matrix in $V = M_{3\times 3}(\mathbb{R})$, and for a positive integer k, denote by A^k the product of A with itself k times. The set $S = \{A, A^2, A^3, \ldots, A^{10}\}$ is a linearly dependent subset of V.

Math 24

1 February 2001
First Hour Exam
(In class part)

NAME (Print!): ______________________________

Problem	Points	Score
1	10	
2	40	
3	10	
4	10	
5	10	
6	20	
Total	100	