Solving PDE's

Remember we can use separation of variables to solve homogeneous (BC) problems.

1- Are the Boundary conditions (BC) homogeneous?

1a- If so, skip to

2- If not, let $u(x,t) = v(x) + w(x,t)$.

Goal: Make a homogeneous BC. problem for $w(x,t)$.

3- Plug into PDE to make differential equation for $v(x)$.

Always keep goal in mind.

4- Make Boundary conditions for $w(x,t)$ & $v(x)$.

(Note: $w(x,t)$ should have zero BC).

5- ~~Trans~~ Create Initial condition for $w(x,t)$. (Note: It will involve $v(x)$)

6- write the 2 problems you need to solve.

(*) DE for $v(x)$ w/ BC

(**) homogeneous PDE

Ex (From Last Term Final)

$$\begin{cases} u_t = u_{xx} & 0 \le x \le \pi \\ u_x(0,t) = u_x(\pi,t) = 10 \\ u(x,0) = \cos(3x) + 10x \end{cases}$$

This problem does not have homogeneous BC.

let $u(x,t) = v(x) + w(x,t)$

Goal make $w(x,t)$ satisfy a homogeneous BC problem. (Heat problem)

Plug into PDE $u_t = u_{xx}$

$$w_{tt} = v'' + w_{xx}$$

$\rightarrow$ let $w_{tt} = w_{xx}$, $v''(x) = 0$.

Now BC.

$$u_x(x,t) = v'(x) + w_x(x,t)$$

$$u_x(0,t) = v'(0) + w_x(0,t) = 10$$

Want $w_x(0,t) = 0 \rightarrow v'(0) = 10$.

likewise $w_x(\pi,t) = 0 \rightarrow v'(\pi) = 10$.

Now make IC. for $w(x,t)$.

$$u(x,0) = v(x) + w(x,0) = 10x + \cos(3x)$$

$$\rightarrow w(x,0) = \cos(3x) - v(x) + 10x$$

This means we need to solve 2 problems

$$v''(x) = 0$$

$$v'(0) = v'(\pi) = 10$$

$$w_{tt} = w_{xx}$$

$$w_x(0,t) = w_x(\pi,t) = 0$$

$$w(x,0) = \cos(3x) + 10x - v(x).$$

We know how to solve each of these problems.