Worksheet #8

(1) Solve the intial value problem. Describe the behavior of the solution as $t \to \infty$.

$$9y'' - 12y' + 4y = 0, \qquad y(0) = 2, \qquad y'(0) = -1$$

1- Characteristic eqn

$9r^2 - 12r + 4 = 0$

2- Find roots

$r = \frac{12 \pm \sqrt{144 - 4(4)(9)}}{2(9)} = \frac{12}{18} = \frac{2}{3}$

3- Soln $y(t) = C_1 e^{2/3t} + C_2 t e^{2/3t}$

$y'(t) = \frac{2}{3} C_1 e^{2/3t} + C_2 \left(\frac{2}{3} t e^{2/3t} + e^{2/3t}\right)$

$y(0) = C_1 = 2$

$y'(0) = \frac{2}{3} C_1 + C_2 = -1 \to C_2 = -1 - \frac{4}{3} = -\frac{7}{3}$

$\to y(t) = 2e^{2/3t} - 7/3\, t e^{2/3t} = (2 - 7/3\, t) e^{2/3t} \to -\infty$ as $t \to \infty$.

(2) Use the method of reduciton of order to find a second solution of the following differential equation:

$$t^2 y'' - 4ty' + 6y = 0, \qquad t > 0,$$

where the first solution is $y_1(t) = t^2$.

Guess $y_2(t) = v(t) t^2$

$y_2'(t) = 2vt + t^2 v'$

$y_2''(t) = 2v + 2tv' + t^2 v'' + 2tv' = t^2 v'' + 4tv' + 2v$

Pluggin this into the DE, we find

$\to t^2(2v + 4tv' + t^2 v'') - 4t(2vt + t^2 v') + 6vt^2 = 0$

Collect like terms.

$t^4 v'' + (4t^3 - 4t^3) v' + (2t^2 - 8t^2 + 6t^2) v = 0$

$\to v'' = 0 \to v(t) = C_1 t + C_2$

$\to y_2(t) = C_1 t^3 + \underbrace{C_2 t^2}_{= y_1(t)}$

$\to y_2(t) = t^3$.