Worksheet #5

(1) Determine if the equation is exact. If it is, find the solution.

(a) $(2x + 4y) + (2x - 2y)y' = 0$

$M = 2x + 4y \qquad N = 2x - 2y$

$M_y = 4 \neq N_x = 2 \Rightarrow$ Not exact

(b) $(e^x \sin y - 2y \sin x)dx + (e^x \cos y + 2\cos x)dy = 0$

$M = e^x \sin y - 2y\sin x \qquad N = (e^x \cos y + 2\cos x)$

$M_y = e^x \cos y - 2\sin x = N_x = e^x \cos y - 2\sin x$ ✓ $\Rightarrow$ exact.

let $\psi_x = M = e^x \sin y - 2y\sin x \Rightarrow \psi(x,y) = e^x \sin y + 2y\cos x + h(y)$

Find $h(y)$: $\psi_y = e^x \cos y + 2\cos x + h'(y) = N = e^x \cos y + 2\cos x$

$\Rightarrow h'(y) = 0.$

$\Rightarrow \psi(x,y) = e^x \sin y + 2y\cos x = C$ is the soln.

(2) Show that the equation

$$\left(\frac{\sin y}{y} - 2e^{-x}\sin x\right)dx + \left(\frac{\cos y + 2e^{-x}\cos x}{y}\right)dy = 0$$

is not exact but becomes exact when multiplied by $\mu(x,y) = ye^x$. Then solve the equation.

$M = \frac{\sin y}{y} - 2e^{-x}\sin x \qquad N = \frac{\cos y + 2e^{-x}\cos x}{y}$

$M_y = -\frac{\sin y}{y^2} + \frac{\cos y}{y} \neq N_x = \frac{1}{y}(-2e^{-x}\sin x - 2e^{-x}\cos x)$

$\Rightarrow$ Not exact.

To show the multiplication of the DE by μ makes the eqn exact, we need to show.

$(\mu M)_y = (\mu N)_x.$

$$\mu M = ye^x\left(\frac{\sin y}{y} - 2e^{-x}\sin x\right) = \sin y\, e^x - 2y\sin x$$

$$\mu N = ye^x\left(\frac{\cos y + 2e^{-x}\cos x}{y}\right) = e^x\cos y + 2\cos x.$$

$$(\mu M)_y = \cos y\, e^x - 2\sin x$$

$$(\mu N)_x = e^x\cos y - 2\sin x$$

These expressions are equal so The new DE is exact.