Worksheet #3

Solve the given initial value problem.

(1) $y' + 2ty = 2te^{-t^2}$, $\quad y(0) = 1$.

1- find $\mu(t)$. $\quad \mu(t) = e^{\int 2t\,dt} = e^{t^2}$

2- multiply $\quad e^{t^2} y' + 2te^{t^2} y = 2t$

3- Rewrite $\quad \frac{d}{dt}(e^{t^2} y) = 2t$

4- Integrate $\quad e^{t^2} y = t^2 + C \rightarrow y(t) = (t^2 + C)e^{-t^2}$

$y(0) = C = 1 \quad \rightarrow \boxed{y(t) = (t^2+1)e^{-t^2}}$

(2) $t^3y' + 4t^2y = e^{-t}$, $\quad y(-1) = 0$, $t < 0$.

Rewrite to use Integrating factor.

$y' + \frac{4}{t} y = e^{-t} t^{-3}$

1- Find $\mu(t) = e^{\int 4/t\,dt} = e^{4\ln t} = e^{\ln t^4} = t^4$

2- multiply $\quad t^4 y' + 4t^3 y = te^{-t}$

3- rewrite $\quad \frac{d}{dt}(t^4 y) = te^{-t}$

continued on next page.

(3) $y' = (1 - 2x)y^2$, $\quad y(0) = -1/6$.

This problem is separable.

1- Separate $\quad \frac{dy}{y^2} = (1-2x)dx$

2- Integrate $\quad -\frac{1}{y} = x - x^2 + C$

$\rightarrow \quad y(x) = \frac{+1}{x^2 - x + C}$

Now solve for c.

$y(0) = \frac{1}{c} = \frac{-1}{6} \rightarrow c = -6$

$\boxed{y(x) = \frac{1}{x^2 - x - 6}}$

(4) $y' = \dfrac{2x}{y + x^2y}$, $\quad y(0) = -2$.

we can separate the variables by factoring.

$y' = \frac{2x}{y(1+x^2)}$

1- Separate $\quad y\,dy = \frac{2x}{1+x^2}dx$

2- integrate $\quad \frac{y^2}{2} = \int \frac{2x}{1+x^2}dx \quad$ u-sub $\quad$ let $u = x^2+1$, $du = 2x\,dx$

$= \int \frac{1}{u} du = \ln u + C$

2 continued

4 - Integrate.

$$t^4 y = \int t e^{-t} dt$$

Recall Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

$u = t \qquad v = -e^{-t}$

$du = dt \qquad dv = e^{-t} dt$

$$\Rightarrow t^4 y(t) = -te^{-t} + \int e^{-t} dt = -te^{-t} - e^{-t} + C$$

$$\Rightarrow y(t) = \left[e^{-t}(t-1) + C\right] t^{-4}$$

$$y(-1) = \left[e(-2) + C\right] = 0 \Rightarrow C = 2e$$

$$\boxed{y(t) = \left[e^{-t}(t-1) + 2e\right] t^{-4}}$$

4 - continued

$$\frac{y^2}{2} = \ln(1+x^2) + C$$

Find C using I.C. $y(0) = -2$

$$\frac{(-2)^2}{2} = \cancel{\ln(1)}^{0} + C = 2$$

$$\frac{y^2}{2} = \ln(1+x^2) + 2$$

$$\Rightarrow y^2 = 2\ln(1+x^2) + 4$$

$$y = \pm\sqrt{2\ln(1+x^2) + 4}$$

We take $-$ to satisfy the I.C.

$$\Rightarrow y(x) = -\sqrt{2\ln(1+x^2)+4}$$