Worksheet #2~~1~~2

(1) Solve the boundary value problem

$$y'' + 4y = \cos x \qquad y(0) = 0, \quad y(\pi) = 0$$

1st Find homogeneous soln.

$r^2+4=0 \Rightarrow r = \pm 2i$

$\Rightarrow y_h(x) = c_1\cos(2x) + c_2\sin(2x)$

Particular soln $y_p(x) = A\cos x + B\sin x \qquad y_p'' = -A\cos x + -B\sin x$

Plug into DE. $(-A+4A)\cos x + (-B+4B)\sin x = \cos x$

$\Rightarrow A = 1/3 \quad B = 0.$

$y(x) = \frac{1}{3}\cos x + c_1\cos(2x) + c_2\sin(2x)$

$y(0) = 1/3 + c_1 = 0 \Rightarrow c_1 = -1/3$

$\Rightarrow y(x) = 1/3\cos x - 1/3\cos(2x) + c_2\sin(2x)$ / $y(\pi) = -\frac{1}{3} - \frac{1}{3} + 0 = 0$

This is not true Thus there is no soln.

(2) Find the eigenvalues and eigenfunctions for

$$y'' + \lambda y = 0 \qquad y(0) = 0, \quad y'(\pi) = 0$$

1st look at characteristic eqn.

$r^2 + \lambda = 0 \Rightarrow r = \pm\sqrt{\lambda}\, i$

$\Rightarrow$ General solution is $y(x) = A\cos(\sqrt{\lambda}\, x) + B\sin(\sqrt{\lambda}\, x)$

$y(0) = A = 0$

$\Rightarrow y(x) = B\sin(\sqrt{\lambda}\, x)$

$y'(x) = B\sqrt{\lambda}\cos(\sqrt{\lambda}\, x)$

$y'(\pi) = B\sqrt{\lambda}\cos(\sqrt{\lambda}\,\pi) = 0$

$\cos\theta = 0$ when $\theta = \frac{(2n-1)\pi}{2}$ n positive integer.

$\Rightarrow \sqrt{\lambda}\,\pi = \frac{2n-1}{2}\pi \Rightarrow \sqrt{\lambda} = \frac{2n-1}{2}$

$\Rightarrow \lambda = \left(\frac{2n-1}{2}\right)^2$

$\Rightarrow y_n(x) = \cos\left(\frac{2n-1}{2}x\right)$ are the eigenfunctions.