Worksheet #1

(1) Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature (i.e. the temperature of its surroundings).

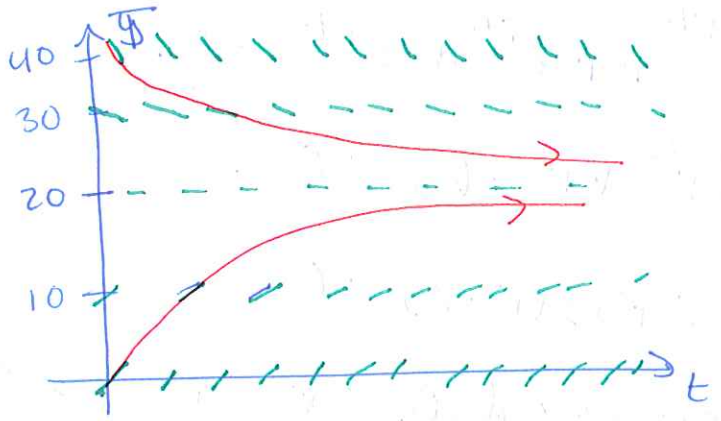
It's the night before Thanksgiving and you realize that you have not defrosted the turkeyyet. When removed from the freezer, the turkey is frozen at $-5°C$. The temperature of the room is $20°C$. Suppose the constant of proportionality is $-\frac{1}{20}$.

(a) Write down a differential equation that models the temperature T of the turkey with respect to time t.

$T' = k(T - T_a)$ T = temp T_a = atmospheric temp.

$$\begin{cases} T' = -\frac{1}{20}(T-20) \\ T(0) = -5 \end{cases}$$

(b) Draw the direction field for the differential equation. By the direction field, what do you expect the temperature T to be as $t \to \infty$?



t	T	$\frac{dT}{dt}$
	20	0
	10	1/2
	0	1
	30	-1/2
	40	-1

as $t \to \infty$, $T \to 20$.

This is what we expect.

(2) Determine the order of the differential equation and state whether or not it is linear.

(a) $\frac{d^2y}{dt^2} + \sin(t+y) = \sin t$

2nd order non-linear. since $\sin(t+y)$ is not linear wrt y

(b) $t^2\frac{dy}{dt} + 4ty = e^t$

1st order linear.

(3) Determine the values of r for which $y = e^{rt}$ is a solution

$$y'' + y' - 6y = 0.$$

Plug $y = e^{rt}$ into the DE.

$$r^2e^{rt} + re^{rt} - 6e^{rt} = 0.$$

factor $e^{rt}(r^2 + r - 6) = 0$.

$e^{rt}(r+3)(r-2) = 0$.

e^{rt} never equals 0. so $r = -3$ & $r = 2$ are the only values of r for which $y = e^{rt}$ is a solution.