

Worksheet #16

(1) Rewrite the initial value problem as a first order linear equation.

$$y''' - xy = 0, \quad y(0) = 1, \quad y'(0) = 2, \quad y''(0) = 3$$

let $t_1 = y$
 $t_2 = y' = t_1'$
 $t_3 = y'' = t_2'$
 $\rightarrow t_3' = y''' = xy$ by DE.
 $= x t_1$

$\rightarrow \begin{cases} t_1' = t_2 \\ t_2' = t_3 \\ t_3' = x t_1 \end{cases}$ is the first order system.
 Now the initial values. $t_1(0) = 1$
 $t_2(0) = 2$
 $t_3(0) = 3$

(2) Let $A = \begin{bmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{bmatrix}$ and $B = \begin{bmatrix} i & 3 \\ 2 & -2i \end{bmatrix}$. Find
 (a) $A - 2B$

$$\begin{bmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{bmatrix} + \begin{bmatrix} -2i & -6 \\ -4 & +4i \end{bmatrix} = \begin{bmatrix} 1-i & -7+2i \\ -1+2i & 2+3i \end{bmatrix}$$

(b) AB

$$\begin{bmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{bmatrix} \begin{bmatrix} i & 3 \\ 2 & -2i \end{bmatrix} = \begin{bmatrix} (1+i)i + 2(-1+2i) & (1+i)3 + (-1+2i)(-2i) \\ (3+2i)i + (2-i)2 & (3+2i)3 + (2-i)(-2i) \end{bmatrix}$$

$$= \begin{bmatrix} -1-2+4i & 3+3i+2i+4 \\ 3i-2+4-2i & 9+6i-4i-2 \end{bmatrix} = \begin{bmatrix} -3+5i & 7+5i \\ 2+i & 7+2i \end{bmatrix}$$

(c) BA

$$\begin{aligned} \begin{bmatrix} 1 & 3 \\ 2 & -2i \end{bmatrix} \begin{bmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{bmatrix} &= \begin{bmatrix} 1(1+i) + 3(3+2i) & 1(-1+2i) + 3(2-i) \\ 2(1+i) - 2i(3+2i) & 2(-1+2i) - 2i(2-i) \end{bmatrix} \\ &= \begin{bmatrix} 1-1+9+6i & -1-2i+6-3i \\ 2+2i-6i+4 & -2+4i-4i-2 \end{bmatrix} = \begin{bmatrix} 7i+8 & 4-4i \\ -4i+6 & -4 \end{bmatrix} \end{aligned}$$

(3) Let $x = \begin{bmatrix} i \\ 3 \\ 1+i \end{bmatrix}$ and $y = \begin{bmatrix} 4-i \\ i \\ 4 \end{bmatrix}$. Find

(a) $x^T y$

$$\begin{aligned} \begin{bmatrix} i & 3 & 1+i \end{bmatrix} \begin{bmatrix} 4-i \\ i \\ 4 \end{bmatrix} &= i(4-i) + 3(i) + 4(1+i) \\ &= 4i + 1 + 3i + 4 + 4i \\ &= 5 + 11i \end{aligned}$$

(b) $(x, y) = \bar{x}^T \bar{y}$

$$\begin{aligned} \begin{bmatrix} i & 3 & 1+i \end{bmatrix} \begin{bmatrix} 4+i \\ -i \\ 4 \end{bmatrix} &= i(4+i) - 3i + 4(1+i) \\ &= 4i - 1 - 3i + 4 + 4i \\ &= 5i + 3 \end{aligned}$$