Worksheet #12

linearly

(1) Are $f_1(t) = 2t - 3$, $f_2(t) = t^2 + t + 1$, $f_3(t) = 2t^2 - t$ linerly dependent or linearly independent? If they are linearly dependent, find a linear relation among them.

$k_1 f_1 + k_2 f_2 + k_3 f_3 = 0 \to k_1(2t-3) + k_2(t^2+t+1) + k_3(t^2-t) = 0$

$1 \quad | \quad -3k_1 + k_2 = 0 \quad \to k_2 = 3k_1$

$t \quad | \quad 2k_1 + k_2 - k_3 = 0 \quad \to k_2 + \frac{2}{3}k_2 + k_2 = 0 \to k_2 = 0$

$t^2 \quad | \quad k_2 + k_3 = 0 \quad \to k_2 = -k_3$

$\to k_1 = k_3 = 0.$

$\to$ The functions are linearly independent.

(2) Solve the intial value problem.

$$y''' - y'' + y' - y = 0$$
$$y(0) = 2, \quad y'(0) = -1, \quad y''(0) = -2$$

How does the solution behave as $t \to \infty$?

1- Find characteristic eqn.

$r^3 - r^2 + r - 1 = 0$

2- Find roots. factor by grouping.

$(r^3 - r^2) + (r-1) = 0$

$\to r^2(r-1) + (r-1) = 0 \to (r^2+1)(r-1) = 0.$

$\to r = 1, \pm i$

3 $\to$ General solution.

$y(t) = C_1 e^t + C_2 \cos t + C_3 \sin t$

4 - Use Inital conditions.

$$y'(t) = c_1 e^t - c_2 \sin t + c_3 \cos t$$

$$y''(t) = c_1 e^t - c_2 \cos t - c_3 \sin t$$

$$y(0) = c_1 + c_2 = 2 \quad ①$$

$$y'(0) = c_1 + c_3 = -1 \quad ②$$

$$y''(0) = c_1 - c_2 = -2 \quad ③$$

$$① + ③ \Rightarrow 2c_1 = 0 \Rightarrow c_1 = 0$$

$$\Rightarrow c_2 = 2$$

$$c_3 = -1$$

$$\Rightarrow y(t) = 2\cos t - \sin t.$$

The solotion will continue to ossillate as $t \to \infty$ w/ amplitude $\sqrt{4+1} = \sqrt{5}$.