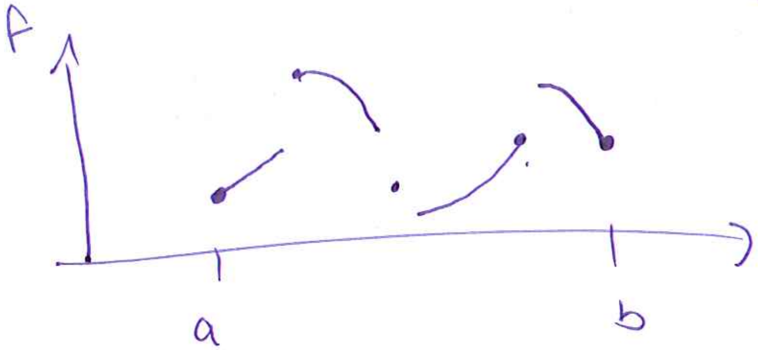
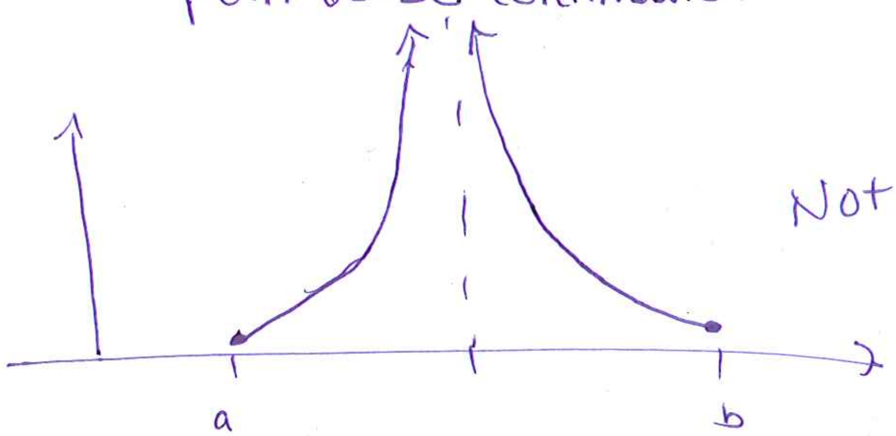
§10.3 Fourier Convergence Thm

Def A function $f(x)$ is said to be piecewise continuous on $a \leq x \leq b$ if

you can partition $a \leq x \leq b$ into finitely many open intervals. & on each of the intervals $f(x)$ is continuous. & the ~~ent~~ limit at the endpoints of each of the intervals is ~~cont~~ finite.



pointwise continuous.



Not pointwise continuous.

<u>Thm</u> Suppose f & f' ~~tx~~ are piecewise continuous on the interval $-L \le x \le L$. Further, suppose that f is defined outside of the interval $-L \le x \le L$ so that it is periodic w/ $2L$ period. Then f has a fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right)$$

The fourier series converges to $f(x)$ at all pts where $f(x)$ is continuous.
& to $\left[\frac{f(x+) + f(x-)}{2}\right]$ at all pts

Where it is discontinuous.

§ 10.H Even & ODD Functions.

Recall • $f(x)$ is an even function if

$f(-x) = f(x)$.

• $f(x)$ is an odd function if

$f(-x) = -f(x)$.

IS the product of two even functions even or odd?

• let f & g be even functions. $p(x) = f(x)g(x)$

$P(-x) = f(-x)\,g(-x) = f(x)g(x) = p(x)$

$\Rightarrow p(x)$ is even.

• let f & g be odd functions, $p(x) = f(x)\,g(x)$.

$P(-x) = f(-x)\,g(-x) = (-1)^2 f(x)g(x) = p(x)$

$\Rightarrow p(x)$ is even

• let ~~f & g be~~ f be even & g be odd.

$P(-x) = f(-x)\,g(-x) = -f(x)\,g(x) = -p(x)$

$\rightarrow P(x)$ is odd.

Integrals.

let $f(x)$ be an odd function on $-L \le x \le L$

$$\int_{-L}^{L} f(x)\,dx = \int_{-L}^{0} f(x)\,dx + \int_{0}^{L} f(x)\,dx$$

let $x = -s$ $dx = -ds$

$$= -\int_{L}^{0} f(-s)\,ds + \int_{0}^{L} f(x)\,dx$$

$$= \int_{L}^{0} f(s)\,ds + \int_{0}^{L} f(x)\,dx$$

$$= -\int_{0}^{L} f(s)\,ds + \int_{0}^{L} f(x)\,dx = 0.$$

let $f(x)$ be an even function on $-L \le x \le L$

$$\int_{-L}^{L} f(x)\,dx = \int_{-L}^{0} f(x)\,dx + \int_{0}^{L} f(x)\,dx$$

let $x = -s$ $dx = -ds$

$$= -\int_{L}^{0} f(-s)\,ds + \int_{0}^{L} f(x)\,dx$$

$$= -\int_{L}^{0} f(s)\,ds + \int_{0}^{L} f(x)\,dx$$

$$= 2\int_{0}^{L} f(x)\,dx.$$

What are the coefficients of ~~an~~ the fourier series of an odd function?

let $f(x)$ be odd function on $-L \le x \le L$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx \;\cancel{=0} = 0.$$

$$a_n = \frac{1}{L}\int_{-L}^{L} \underbrace{f(x)\cos\left(\frac{n\pi x}{L}\right)}_{\text{odd}} dx = 0.$$

$$b_n = \frac{1}{L}\int_{-L}^{L} \underbrace{f(x)\sin\left(\frac{n\pi x}{L}\right)}_{\text{even}} dx \;\cancel{= \frac{2}{L}\int_0^L f(x)\sin}$$

$$= \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$\rightarrow f(x) = \cancel{\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)}$$

$$= \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

let f(x) be an even function on $-L \le x \le L$

Then

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx = \frac{2}{L}\int_{0}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} \underbrace{f(x)\cos\left(\frac{n\pi x}{L}\right)}_{\text{even}} dx = \frac{2}{L}\int_{0}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right) dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} \underbrace{f(x)\sin\left(\frac{n\pi x}{L}\right)}_{\text{odd}} dx = 0.$$

$$\to f(x) = \cancel{\sum_{n}} \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$