

Assignment 1

1. (taken from Lebl's notes) Consider the differential equation

$$(y')^2 = 1 - y^2.$$

Is $y = \sin(t)$ a solution? Is $y = \cos(t)$ a solution? Is $y = \sin(t) + \cos(t)$ a solution?

2. Find all values (if any) of r such that e^{rt} is a solution of $y'' + 5y' + 4y = 0$.
3. Consider the differential equation

$$y' = y - y^3$$

- (a) Draw the direction field. (You may use software if desired.)
 - (b) Find the equilibrium solutions.
 - (c) Describe the behavior of the solutions as $t \mapsto \infty$. Note that you will have different behavior depending on the initial value of y at time $t = 0$. Indicate explicitly how this behavior depends on the initial condition. (You are to carry out the same procedure here as in problems 11-14 in section 1.1 of the text.)
4. Section 1.1: 21 (The solution is in the text, but you are expected to explain the reasoning that leads to the solution.)
 5. In the previous problem concerning the amount of chemical in a pond, suppose in addition to the flow into and out of the pond described in part (a), that pure water is evaporating from the pond at the rate of one gallon per hour. Write down a differential equation for the amount of chemical in the pond at any given time.
 6. Indicate whether each of the following differential equations is linear:
 - (a) $e^t y'' + t y' + 2 = 0$.
 - (b) $y y' + 2 = 0$