**Dartmouth College**
Mathematics 23 - Assignment 25

**Note.** This assignment will *not* be collected. However, it covers Section 10.7, which you are responsible for on the exam.

1. Boyce and DiPrima Section 10.7: 4 (a). (You are encouraged to do (b)–(e) if you have an appropriate software package that you are comfortable with.)

2. Boyce and DiPrima Section 10.7: 8(a). Again (b–e) optional.

3. Use separation of variables to find the solution $u(x,t)$ for $x \in [0,1]$ and $t > 0$ of the following problem:

$$u_{xx} + tu_t = 0,\ \ u(0,t) = 0,\ \ u(1,t) = 0,\ \ u(x,1) = 1 - x.$$

4. (Travelling waves.) Consider the wave equation $u_{tt} = a^2 u_{xx}$ on the entire real line. Show by direct computation that if $f(x)$ is any twice-differentiable function on $\mathbb{R}$, then the functions $v(x,t) = f(x-at)$ and $w(x,t) = f(x+at)$ both satisfy the wave equation. (This is discussed in the text, but check directly.) Thus $u(x,t) = f(x-at) + f(x+at)$ is also a solution. (One can also consider functions $f$ that are just piecewise smooth.)

5. Boyce and DiPrima 10.7: 16(c,d).

   **Remark. Travelling waves and stationary, or standing, waves.** When we solved the wave equation on an interval $[0,L]$ in class, we expressed the solution as an infinite sum, e.g, in the case of zero initial velocity, we obtained

$$u(x,t) = \sum_{n=1}^{\infty} \sin(\frac{n\pi x}{L}) \cos(\frac{n\pi at}{L}).$$

   Each term is called a stationary or standing wave of frequency $\frac{n\pi a}{L}$. This looks quite different from the traveling wave solutions in the previous two problems. However, (as discussed in the text), this solution $u(x,t)$ can also be expressed as

$$u(x,t) = \frac{1}{2}(h(x-at) + h(x+at)$$

   where $h$ is the odd periodic extension of period $2\pi$ of the initial function $u(x,0) = f(x)$.