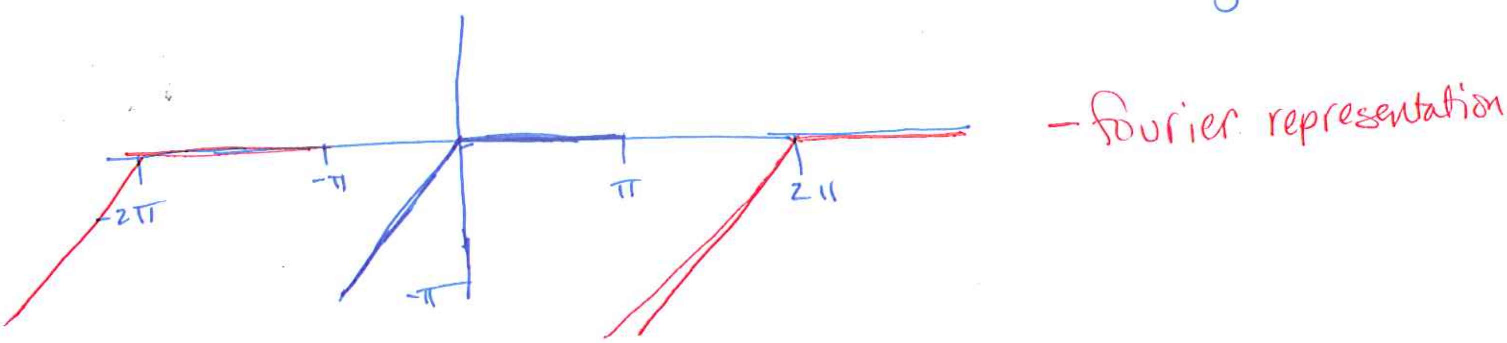
Ex Find the Fourier Series representation of

$$f(x) = \begin{cases} x & \text{for } -\pi \le x \le 0 \\ 0 & \text{for } 0 \le x \le \pi. \end{cases}$$

The Fourier series will be 2π periodic.

What will it look like



What is L? $L = \pi$.

Now to make the coefficients.

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx = \frac{1}{\pi}\frac{x^2}{2}\Big|_{-\pi}^{0} = \left(0 - \frac{\pi^2}{2}\right)\frac{1}{\pi} = -\frac{\pi^2}{2}\left(\frac{1}{\pi}\right) = -\frac{\pi}{2}$$

$$a_m = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{m\pi x}{L}\right)dx = \frac{1}{L}\int_{-\pi}^{0} x\cos(mx)\,dx$$

$u = x$, $du = dx$, $v = \frac{1}{m}\sin(mx)$, $dv = \cos mx\,dx$

$$= \frac{1}{L}\Bigg[\underbrace{\frac{x}{m}\sin(mx)\Big|_{-\pi}^{0}}_{=0} - \int_{-\pi}^{0}\frac{1}{m}\sin(mx)\,dx\Bigg]$$

$$= \frac{1}{L}\cdot\left(\frac{-1}{m^2}\cos(mx)\Big|_{-\pi}^{0}\right) = \frac{1}{L}\frac{1}{m^2}\left(1 - \underbrace{\cos(m\pi)}_{=(-1)^m}\right)$$

$$= \frac{1}{m^2 L}\left(1 - (-1)^m\right) = \begin{cases} 0 & \text{if } m \text{ is even} \\ \frac{2}{\pi m^2} & \text{if } m \text{ is odd.} \end{cases}$$

Now to find b_m's.

$$b_m = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{m\pi x}{L}\right)dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin(mx)\,dx$$

$u = x$ $\quad v = \frac{1}{m}\cos(mx)$

$du = dx$ $\quad dv = \sin(mx)dx$

$$= \frac{1}{\pi}\left(\frac{x}{m}\cos(mx)\Big|_{-\pi}^{0} + \frac{1}{m}\int_{-\pi}^{0}\cos(mx)\,dx \right)$$

$$= \frac{1}{\pi}\left(0 + \left(\frac{-\pi}{m}\cos(m\pi)\right) + \underbrace{\frac{1}{m^2}\sin(mx)\Big|_{-\pi}^{0}}_{=0} \right)$$

$$= \frac{1}{m}(-1)^{m+1}$$

$\Rightarrow$ The fourier series is

$$f(x) \approx \frac{-\pi}{4} + \sum_{m=1}^{\infty}\left[\frac{2\cos((2m-1)x)}{\pi(2m-1)^2} - \frac{(-1)^{m+1}}{m}\sin(mx) \right]$$