Math 23, Spring 2007

Lecture 6

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Outline

Last class

Today's material

- Second order equations
- Linear second order equations
- Qualitative behavior

Group work

Next class

Material from last class

- Models via first order equations
- Refinement and augmentation

General second order equations

The most general second order equation is of the form:

$$\frac{d^2y}{dt^2} = f(t, y, y')$$

Such an equation i s said to be linear if

$$f(t, y, y') = g(t) - p(t)y' - q(t)y$$

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Linear second order equations

Two forms

$$y'' + p(t)y' + q(t)y = g(t)$$

or

$$P(t)y'' + Q(t)y' + R(t)y = g(t)$$

These are equivalent when $P(t) \neq 0$.

If $g(t) = 0$, the equation is called *homogeneous*.

Otherwise, it is called *inhomogeneous*.

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Linear homogeneous second order equations

Solutions for constant coefficients

Assuming constant coefficients:

$$ay'' + by' + cy = 0$$

To find solutions, we look at functions of the form $y(t) = e^{rt}$

$$ar^2e^{rt} + bre^{rt} + ce^{rt} = 0$$

Since $e^{rt} \neq 0$ we have the *characteristic equation*:

$$ar^2 + br + c = 0$$

which can be solved using the quadratic equation:

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Linear homogeneous second order equations

Solutions for constant coefficients

Example

$$y'' + y' - 2y = 0,\ y(0) = 1, y'(0) = 1$$

Linear homogeneous first order equations

Qualitative behavior

$r_1, r_2 < 0$

The limit of y as $t \to \infty$ is zero

$r_1, r_2 > 0$

The limit of y as $t \to \infty$ is $\pm\infty$

$r_1 > 0, r_2 < 0$

The limit of y as $t \to \infty$ is $\pm\infty$. There could be an interaction between the two solutons for small t.

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Examples:

1. $$y'' + 4y' + 3y = 0,\ y(0) = 2, y'(0) = -1$$

2. $$2y'' + y' - 4y = 0,\ y(0) = 0, y'(0) = 1$$

3. $$4y'' - y = 0,\ y(-2) = 1, y'(-2) = -1$$

Work for next class

- Reading: 3.2
- xhour tomorrow on elementary linear algebra. Get the handout from our website
- Homework 3 is due monday 4/16