Math 23, Spring 2007

Lecture 13

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Outline

Midterm results

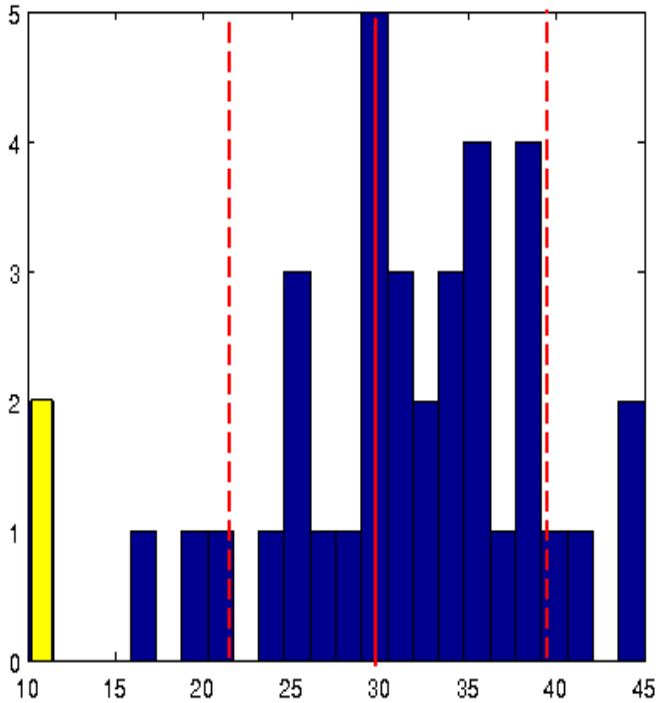

Figure: mean = 31, standard deviation = 8

Material from last class

- Series solutions for second order linear ODE
- $$y = \sum_{n=0}^{\infty} a_n (t - t_0)^n$$

Example from last class

$$y'' + ty = 0$$

Example from last class

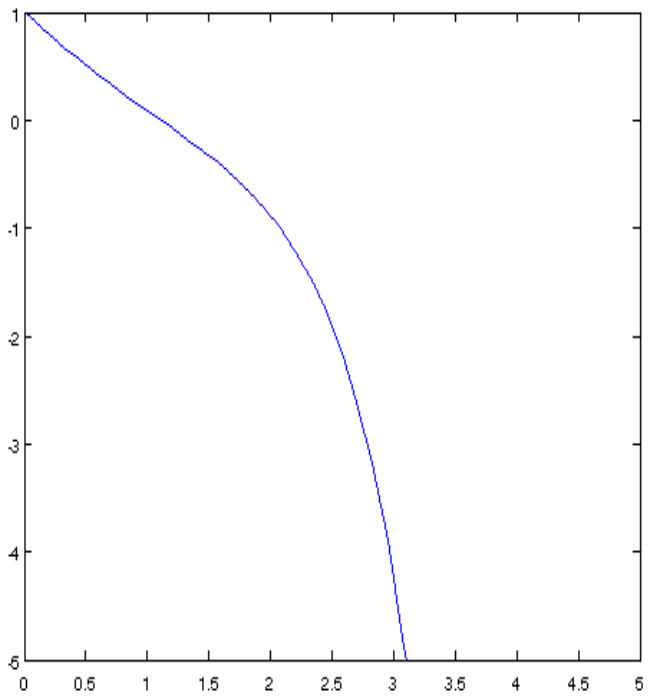

Figure: A plot of the approximate solution

A variation

$$y'' + \sin(t)y = 0$$

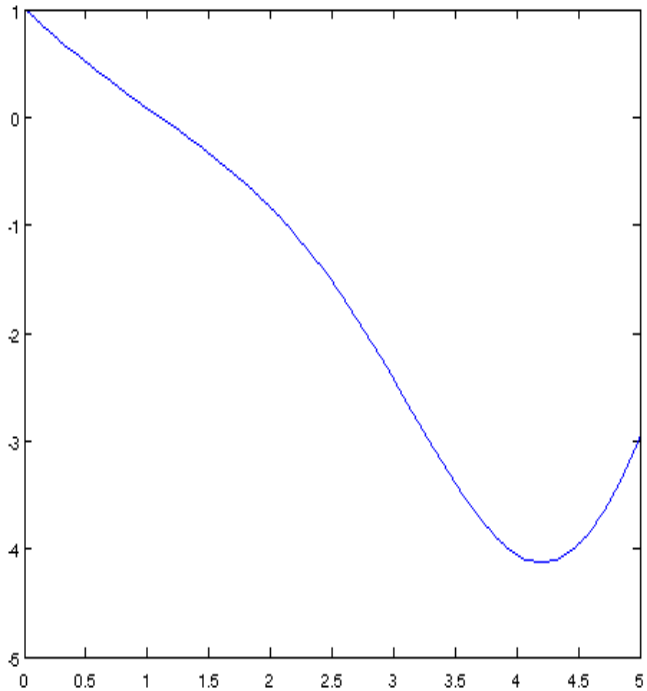

Figure: A plot of the approximate solution

Theorem

Theorem

Consider the equation

$$P(x)y'' + Q(x)y' + R(x)y = 0$$

If x_0 is an ordinary point, i.e. $p = Q/P$ and $q = R/P$ are analytic at x_0 then the general solution of the ODE is

$$y = \sum_{n=0}^{\infty} a_n(x - x_0)^n = a_0 y_1(x) + a_1 y_2(x)$$

where a_0, a_1 are arbitrary and y_1 and y_2 are linearly independent series solutions that are analytic at x_0. Moreover the radii of convergence of the y_i are at least as large as the minimum of the radii of convergence of p and q.

Example

Legendre's equation:

$$(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$$

Work for next class

- Read: 5.1-5.3
- Homework 5 is due wednesday 5/1