

Math 23, Spring 2007

Lecture 13

Scott Pauls ¹

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4/25/07

Outline

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2007

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In class midterm results

In class midterm
results

Last class

Last class

Today's material

Series solutions around ordinary points

Today's material
Series solutions around
ordinary points

Next class

Next class

Midterm results

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In class midterm
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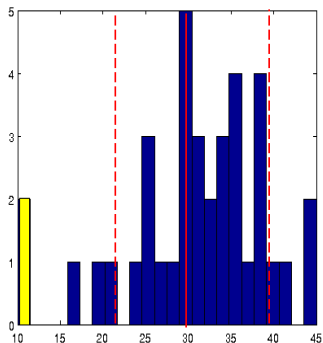


Figure: mean = 31, standard deviation = 8

- ▶ Series solutions for second order linear ODE



$$y = \sum_{n=0}^{\infty} a_n (t - t_0)^n$$

Example from last class

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$$y'' + ty = 0$$

Example from last class

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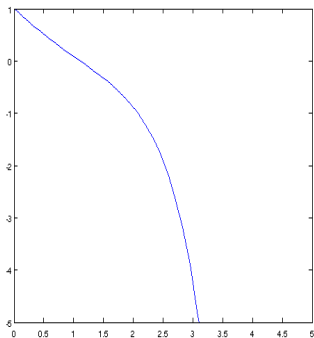


Figure: A plot of the approximate solution

A variation

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$$y'' + \sin(t)y = 0$$

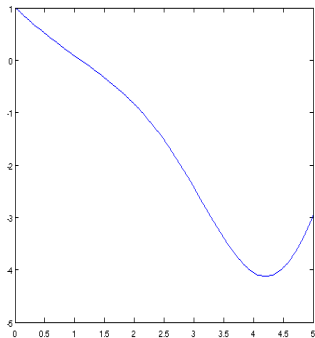


Figure: A plot of the approximate solution

Theorem

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Theorem

Consider the equation

$$P(x)y'' + Q(x)y' + R(x)y = 0$$

If x_0 is an ordinary point, i.e. $p = Q/P$ and $q = R/P$ are analytic at x_0 then the general solution of the ODE is

$$y = \sum_{n=0}^{\infty} a_n(x - x_0)^n = a_0y_1(x) + a_1y_2(x)$$

where a_0, a_1 are arbitrary and y_1 and y_2 are linearly independent series solutions that are analytic at x_0 . Moreover the radii of convergence of the y_i are at least as large as the minimum of the radii of convergence of p and q .

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Example

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Legendre's equation:

$$(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$$

Work for next class

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Next class

- ▶ Read: 5.1-5.3
- ▶ Homework 5 is due wednesday 5/1