Chapter 7. Eigenvalues and eigenvectors

I. The following list of commands imitate somehow what one does when computes on paper (without a computer) the eigenvalues adnn values of a matrix.

```
> restart;
> with(linalg):
Warning, the protected names norm and trace have been redefined and unprotected
> A:=Matrix([[3,2,4],[2,0,2],[4,2,3]]);
```

$$A := \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

```
> Id:=Matrix(3,3,shape=identity):Ar:=A-r*Id:evalm(Ar);
```

$$\begin{bmatrix} -r+3 & 2 & 4 \\ 2 & -r & 2 \\ 4 & 2 & -r+3 \end{bmatrix}$$

```
> Ar:=matrix([[3-r,2,4],[2,-r,2],[4,2,3-r]]);
```

$$Ar := \begin{bmatrix} -r+3 & 2 & 4 \\ 2 & -r & 2 \\ 4 & 2 & -r+3 \end{bmatrix}$$

```
> det(Ar);
```

$$-r^3 + 6\,r^2 + 15\,r + 8$$

```
> factor(%);
```

$$-(r-8)\,(r+1)^2$$

This shows that the eigenvalues of A are r=8 and r=-1 (double).

-----------------*The eigenvalues for r=8:*

```
> A8:=subs(r=8,evalm(Ar));
```

$$A8 := \begin{bmatrix} -5 & 2 & 4 \\ 2 & -8 & 2 \\ 4 & 2 & -5 \end{bmatrix}$$

```
> x:=[xi1*exp(8*t),xi2*exp(8*t),xi3*exp(8*t)];
```

$$x := [\xi1\,\mathbf{e}^{(8\,t)}, \xi2\,\mathbf{e}^{(8\,t)}, \xi3\,\mathbf{e}^{(8\,t)}]$$

```
> A8x:=multiply(A8,x);
```

$$A8x := [-5\,\xi1\,\mathbf{e}^{(8\,t)} + 2\,\xi2\,\mathbf{e}^{(8\,t)} + 4\,\xi3\,\mathbf{e}^{(8\,t)}, 2\,\xi1\,\mathbf{e}^{(8\,t)} - 8\,\xi2\,\mathbf{e}^{(8\,t)} + 2\,\xi3\,\mathbf{e}^{(8\,t)}, 4\,\xi1\,\mathbf{e}^{(8\,t)} + 2\,\xi2\,\mathbf{e}^{(8\,t)} - 5\,\xi3\,\mathbf{e}^{(8\,t)}]$$

```
> solve({A8x[1]=0,A8x[2]=0,A8x[3]=0},{xi1,xi2,xi3});
```

$$\{\xi2 = \xi2, \xi1 = 2\,\xi2, \xi3 = 2\,\xi2\}$$

----------------*The eigenvectors for r=-1:*

```
> A1:=subs(r=-1,evalm(Ar));
```

$$A1 := \begin{bmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{bmatrix}$$

```
> A1x:=multiply(A1,x);
```

$A1x :=$

$$[4\,\xi1\,\mathbf{e}^{(8\,t)} + 2\,\xi2\,\mathbf{e}^{(8\,t)} + 4\,\xi3\,\mathbf{e}^{(8\,t)},\ 2\,\xi1\,\mathbf{e}^{(8\,t)} + \xi2\,\mathbf{e}^{(8\,t)} + 2\,\xi3\,\mathbf{e}^{(8\,t)},\ 4\,\xi1\,\mathbf{e}^{(8\,t)} + 2\,\xi2\,\mathbf{e}^{(8\,t)} + 4\,\xi3\,\mathbf{e}^{(8\,t)}]$$

```
> solve({A1x[1]=0,A1x[2]=0,A1x[3]=0},{xi1,xi2,xi3});
```

$$\{\xi3 = \xi3,\ \xi1 = \xi1,\ \xi2 = -2\,\xi1 - 2\,\xi3\}$$

So we get (1,-2,0) and (0,-2,1) as linearly independent eigenvectors.

Check next the independence of the solutions that are obtained from these eigenvectors:

```
> W:=matrix([[exp(8*t),1/2*exp(8*t),exp(8*t)],[exp(-t),-2*exp(-t),0]
  ,[0,-2*exp(-t),exp(-t)]]);
```

$$W := \begin{bmatrix} \mathbf{e}^{(8\,t)} & \frac{1}{2}\,\mathbf{e}^{(8\,t)} & \mathbf{e}^{(8\,t)} \\ \mathbf{e}^{(-t)} & -2\,\mathbf{e}^{(-t)} & 0 \\ 0 & -2\,\mathbf{e}^{(-t)} & \mathbf{e}^{(-t)} \end{bmatrix}$$

```
> det(W);
```

$$-\frac{9}{2}\,\mathbf{e}^{(8\,t)}\,(\mathbf{e}^{(-t)})^2$$

II. As usually Maple has its own way of doing things :O)

```
> restart:with(LinearAlgebra):
> Eigenvalues(A);
```

$$\begin{bmatrix} 8 \\ -1 \\ -1 \end{bmatrix}$$

```
> Eigenvectors(A);
```

$$\begin{bmatrix} -1 \\ -1 \\ 8 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 \\ -2 & -2 & \frac{1}{2} \\ 1 & 0 & 1 \end{bmatrix}$$

```
> with(LinearAlgebra):A:=Matrix([[3,2],[2,0]]):Eigenvectors(A);
```

$$\begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

```
>
```