

First Exam

MATH 23 — WINTER 2014

NAME:

SECTION: **11** **12**

This exam has 8 questions on 12 pages, for a total of 200 points.

You have 120 minutes to answer all questions.

This is a closed book exam.

Use of calculators and other electronic devices is not permitted.

Show all your work, justify all your answers.

Question	Points	Score
1	30	
2	20	
3	15	
4	15	
5	35	
6	30	
7	25	
8	30	
Total:	200	

- 30 1. A tank contains 100 gal of water and 50 oz of a chemical. Water containing a concentration of $\frac{1}{4}(1 + \frac{t}{2})$ oz/gal of this chemical flows into the tank at a rate of 2 gal/min, and the mixture flows out at the same rate.
- (a) Write a differential equation for the amount of chemical in the tank at any time.
- (b) Find the amount of chemical in the tank at any time.

20 2. Find the solution of the initial value problem.

$$y' + 2y = te^{-2t}, \quad y(1) = 0$$

15 3. Find the general solution to:

$$\frac{dy}{dx} = \frac{x^2}{y},$$

- 15 4. *WITHOUT FINDING A SOLUTION* determine an interval in which the solution of the initial value problem is guaranteed to exist.

$$(4 - t^2)y' + 2ty = 3t^2, \quad y(1) = -3$$

35 5. Suppose a population y is modelled by the equation

$$y' = -y \left(1 - \frac{y}{a}\right) \left(1 - \frac{y}{1000}\right)$$

(a) For $a = 200$, sketch:

- the graph of y' as a function of y

- the phase line

- several possible solution curves $y(t)$, including any equilibrium solutions.

(b) For arbitrary $a > 0$, characterise the stability of the equilibrium solutions. *Do not assume $a < 1000$.*

(c) Sketch a bifurcation diagram for the parameter a .

30 6. Find the general solution to the following differential equations. *You do not have to justify that your solution is the general solution.*

(a) $y'' - 6y' + 18y = 0$

(b) $4y'' - 4y' + 3y = 0$

- 25 7. (a) Find two constants n such that $y = t^n$ is a solution to the differential equation

$$t^2y'' + 3ty' - 3y = 0$$

- (b) Write down the general solution to the differential equation for $t < 0$ and use the Wronskian to justify that this is the general solution.

- 30 8. Solve the initial value problem using the method of undetermined coefficients.

$$y'' - 2y' + y = 3te^{2t}, \quad y(0) = 2, \quad y'(0) = 4$$

Scrap paper. If you want any work here to be graded, indicate this clearly on the page the question is on.

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