Math 23 Diff Eq: Homework 7

due Wed Nov 16 ... but best if do relevant questions after each lecture

You will do a little Matlab again this time—I have tried to maximize the intuition it gives you but minimize pain (little new coding, but try it early and ask if stuck). In problem "A" you solve the familiar damped oscillator but from a *ODE system* viewpoint. Why bother doing this numerically when in the take-home you already did it analytically? Because most real-world problems are *nonlinear* and *not analytically solvable*: such numerical methods are then among your only friends.

9.1: 17 (reviews mass-spring using your new geometric language. The next problem will help you to check your answers. By the way, nothing electric is required).

A. Matlab's Runge-Kutta solver `ode45` can also handle systems of ODEs, by feeding it a *column vector* initial condition, here use `x0 = [1;0]`, and a column vector function, *e.g.* to solve the equation $\mathbf{x}' = A\mathbf{x}$ with general 2-by-2 matrix[1] use the (t-inpedendent) function `f = @(t,x) A*x`. Use this (cannibalizing `intro.m` or your HW2) to numerically solve the above mass-spring system for $m = 1$, $\gamma = 0.1$, $k = 10$, then plot x vs t in $0 < t < 100$. Then do a 3D plot of $\mathbf{x}(t)$, that is (x, y, t). If you got your output vectors `xs` via `[ts, xs] = ode45(...)`, then you'll want

```
plot3(xs(:,1), xs(:,2), ts); axis vis3d; xlabel('x'); ylabel('y'); zlabel('t');
```

Rotate (click box symbol then drag plot) from all angles until you grasp its shape—cool, eh?

9.2: 3, 5, 6 (use `pplane7` or applet for these two), 19 (just use `pplane7` or the applet to plot solutions as before).

9.3: 1 (write in the form $\mathbf{x}' = A\mathbf{x} + \mathbf{g}(\mathbf{x})$ then show $\mathbf{g}$ is small enough to make the system almost linear, then use A to deduce type and stability),

12 (Look for critical points by sketching the locus of $x' = 0$ in the plane, and of $y' = 0$, then looking for intersections. Make sure you write down the A matrices at both types of critical point—requires thought. The extension of conclusions is important here. You may want to increase the range of `y` values plotted by `pplane7` or applet).

The rest will be less time-consuming:

10.1: 1, 5, 8 (the last two also review your 2nd-order linear ODE methods), 14, 16.

10.2: 8, 10 (both easy), 14 (welcome to your first Fourier series!).

[1] What's A in your case? Remember it will need to be defined *before* `f` is.