MATH 22 REVIEW.

Barnett
8/23/06

A) Find the eigenvalues of $A = \begin{bmatrix} 3 & 0 & 2 \\ 5 & 6 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

Is it diagonalizable? (Why)

B) i) Find bases for the eigenspaces of $A = \begin{bmatrix} 5 & -4 & -2 \\ -4 & 5 & 2 \\ -2 & 2 & 2 \end{bmatrix}$ given the eigenvalues are $\lambda = 1, 10$. Is it diagonalizable?

ii) since A is symmetric, it is orthogonally diagonalizable. Consider $\vec{v}_1, \vec{v}_2$, the basis you found for the $\lambda=1$ eigenspace. Find $\hat{v}_2$, the projection of $\vec{v}_2$ onto Span$\{\vec{v}_1\}$.

iii) Use this to replace $\vec{v}_2$ by an eigenvector (in the $\lambda=1$ eigenspace) which is orthogonal to $\vec{v}_1$

iv) Now you have an orthogonal set. Finally write $A = PDP^T$ where P is an orthogonal matrix: $P = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$ $D = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$

C) Consider vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$. The planes $x=0$ and $z=0$ meet at $90°$. Is one the orthogonal complement of the other? Why?

MATH 22 REVIEW. ① Barnett 5/29/06

A) Find the eigenvalues of $A = \begin{bmatrix} 3 & 0 & 2 \\ 5 & 6 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ Is it diagonalizable? (Why) Yes, since all 3 eigvals distinct.

2 zeros → $\begin{vmatrix} 3-\lambda & 0 & 2 \\ 5 & 6-\lambda & 4 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} 3-\lambda & 0 \\ 5 & 6-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda)(6-\lambda)$ $\quad \lambda = 1, 3, 6.$

B) i) Find bases for the eigenspaces of $A = \begin{bmatrix} 5 & -4 & -2 \\ -4 & 5 & 2 \\ -2 & 2 & 2 \end{bmatrix}$

given the eigenvalues are $\lambda = 1, 10$. Is it diagonalizable?

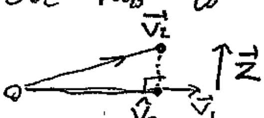
$\lambda=1$: $A - \lambda I = \begin{bmatrix} 4 & -4 & -2 \\ -4 & 4 & 2 \\ -2 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad \vec{x} = x_2 \underbrace{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}_{\vec{v}_1} + x_3 \underbrace{\begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}}_{\vec{v}_2}$

eigenspace basis is $\{\vec{v}_1, \vec{v}_2\}$, $\dim \text{Nul}(A - 1\cdot I) = 2$ equal to algebraic mult. ⇒ diagonalizable

$\lambda = 10$: $A - 10I = \begin{bmatrix} -5 & -4 & -2 \\ -4 & -5 & 2 \\ -2 & 2 & -8 \end{bmatrix} \xrightarrow{\frac{1}{2}R_3} \sim \begin{bmatrix} 1 & -1 & 4 \\ 0 & -9 & 18 \\ 0 & -9 & 18 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad \vec{v}_3 = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$ (3 L.I. eigvecs).

ii) since A is symmetric, it is orthogonally diagonalizable. Consider $\vec{v}_1, \vec{v}_2$, the basis you found for the $\lambda=1$ eigenspace. Find $\hat{v}_2$, the projection of $\vec{v}_2$ onto Span$\{\vec{v}_1\}$. $\quad \hat{v}_2 = \frac{v_2 \cdot v_1}{v_1 \cdot v_1}\vec{v}_1 = \frac{1/2}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/4 \\ 1/4 \\ 0 \end{bmatrix}$

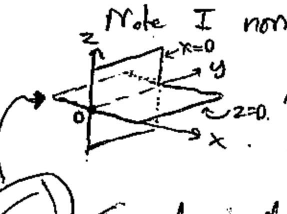
iii) Use this to replace $\vec{v}_2$ by an eigenvector (in the $\lambda=1$ eigenspace) which is orthogonal to $\vec{v}_1$



$\vec{z} = \vec{v}_2 - \hat{v}_2 = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1/4 \\ 1/4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/4 \\ -1/4 \\ 1 \end{bmatrix}$ ← this is $\perp \vec{v}_1$ and in Span$\{\vec{v}_1, \vec{v}_2\}$, therefore a $\lambda=1$ eigvec.

iv) Now you have an orthogonal set. Finally write $A = PDP^T$ where P is an orthogonal matrix: $\quad P = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{18} & -2/3 \\ 1/\sqrt{2} & -1/\sqrt{18} & 2/3 \\ 0 & 4/\sqrt{18} & 1/3 \end{bmatrix}$ $\quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}$

Note I normalized each of $\vec{v}_1, \vec{v}_2$ (new one), $\vec{v}_3$ $\qquad \lambda_1$ (twice), λ_2.



Although these 2 planes meet at $\perp$, they are not each other's orthogonal complement. Eg, the point $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ is in $x=0$ but is not $\perp$ to $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ & $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ which span $z=0$. $W^\perp$ & W must intersect at $\vec{0}$ only!

C) Consider vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$. The planes $x=0$ and $z=0$ meet at $90°$. Is one the orthogonal ...