MATH 22 WORKSHEET : determinants & row operations 7/19/06

A) Compute the determinants

$$\begin{vmatrix} 2 & 7 \\ 0 & 1 \end{vmatrix}$$

(which row or column is easiest to expand)

$$\begin{vmatrix} 3 & -1 & 4 \\ 0 & 2 & 7 \\ 0 & 0 & 1 \end{vmatrix}$$

already know det

$$\begin{vmatrix} -5 & 8 & 10 & 39 \\ 0 & 3 & -1 & 4 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

what can you deduce about det of any upper or lower triangular matrix?

B) Consider $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ?$

By what factor is det changed? FILL IN GENERAL RULE.

row ops.

- Swap rows: $\begin{vmatrix} c & d \\ a & b \end{vmatrix} = ?$
- Scale a row: $\begin{vmatrix} ka & kb \\ c & d \end{vmatrix} = ?$
- Add multiple of row to another: $\begin{vmatrix} a & b \\ c+ka & d+kb \end{vmatrix} =$

these rules also work for general nxn matrices (tricky proof in book p. 173-4)!

C) Use row reduction (while keeping track of above changes to det) to find the determinant:

$$\begin{vmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{vmatrix} = (\text{any factor?}) \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix}$$

(start to row reduce)

$$= (\text{factor?}) \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix} = \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix}$$ etc

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SOLUTIONS

A) Compute the determinants (which row or column is easiest to expand?)

$$\begin{vmatrix} 2 & 7 \\ 0 & 1 \end{vmatrix} = 2\cdot 1 - 7\cdot 0 = 2$$

$$\begin{vmatrix} 3 & -1 & 4 \\ 0 & 2 & 7 \\ 0 & 0 & 1 \end{vmatrix}$$

already know det

Cofactor expansion down 1st col:

$$\det A = 3\cdot\begin{vmatrix} 2 & 7 \\ 0 & 1 \end{vmatrix} + 0 + 0 = 3\cdot 2 = 6$$

$$\begin{vmatrix} -5 & 8 & 10 & 39 \\ 0 & 3 & -1 & 4 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Again, 1st col:

$$\det A = -5\cdot\begin{vmatrix} 3 & -1 & 4 \\ 0 & 2 & 7 \\ 0 & 0 & 1 \end{vmatrix} = -5\cdot 6 = -30.$$

what can you deduce about det of any upper or lower triangular matrix? Here we did upper tri case: ended up product of diagonal entries. Check also for lower.

B) Consider $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ? \; ad - bc$

By what factor is det changed? FILL IN GENERAL RULE.

row ops.

- Swap rows: $\begin{vmatrix} c & d \\ a & b \end{vmatrix} = ? \; cb - da = -(ad-bc)$ — mult. by $-1$
- Scale a row: $\begin{vmatrix} ka & kb \\ c & d \end{vmatrix} = ? \; kad - kbc = k(ad-bc)$ — mult. by $k$
- Add multiple of row to another: $\begin{vmatrix} a & b \\ c+ka & d+kb \end{vmatrix} = ad + akb - bc - bka = ad - bc$ — no change.

These rules also work for general nxn matrices (proof in book p. 173-9) !

C) Use row reduction (while keeping track of above changes to det) to find the determinant:

$$\det A = \begin{vmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{vmatrix} = \begin{vmatrix} 1 & -4 & 2 \\ 0 & 0 & -5 \\ 0 & 3 & 2 \end{vmatrix} = (-1)\begin{vmatrix} 1 & -4 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & -5 \end{vmatrix}$$

any factor? no — start to row reduce

factor? (swap) $-1$

etc

done; is in EF.

Upper-triangular $\Rightarrow \det A = (-1)\cdot(\text{product of diag entries}) = -(1\cdot 3\cdot(-5)) = +15.$