Math 22: Linear Algebra. MIDTERM 1

2 hrs, no calculators. Please answer on this sheet. Your NAME:

1. [10 points] Solve the linear system $A\mathbf{x} = \mathbf{b}$ given

$$A = \begin{bmatrix} 2 & -6 & 1 & -2 \\ -1 & 3 & 2 & 6 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

   (a) If inconsistent, explain why. If consistent, write the general solution in parametric *vector* form (*i.e.* in the form $\mathbf{x} = \mathbf{p} + s\mathbf{u} \cdots$ etc):

$$\mathbf{x} =$$

(b) Write in the same form the solution to the corresponding *homogeneous* problem $A\mathbf{x} = \mathbf{0}$:

$$\mathbf{x} \;=$$

2. [11 points]

(a) [2 points] True/false: if a system of linear equations has two different solutions, then it must have infinitely many solutions.

(b) [3 points] True/false: a set of three vectors in $\mathbb{R}^2$ can be linearly independent. (Explain your answer)

(c) [2 points] A transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$ maps $(1, 0)$ to $(3, 4)$ and $(2, 0)$ to $(6, 7)$. What (if anything) can we say about whether T is a *linear* transformation?

(d) [4 points] Suppose A and B are $n \times n$ matrices, and that B is invertible, and that AB is invertible. Prove that A is also invertible. [Hint: write $C = AB$ then try to construct an inverse of A. Be careful not to assume the inverse of A exists as part of your proof!]

3. [10 points] Consider the three vectors $\begin{bmatrix} 4 \\ 2 \\ -6 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 0 \\ h \end{bmatrix}$,

where h is some number.

(a) Fix $h = 0$. Can the last one of these vectors be written as a linear combination of the first two?

(b) Still fixing $h = 0$, is the set of vectors linearly independent? If not, give a linear dependence relation between the vectors.

(c) For what values of h, if any, do the three vectors span $\mathbb{R}^3$?

4. [11 points] Compute the inverses of the following matrices or explain why they do not exist:

(a) $\begin{bmatrix} 2 & -6 \\ 1 & -3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -2 & 1 \\ -1 & 5 & 6 \\ 5 & -4 & 18 \end{bmatrix}$

5. [9 points] Find the standard matrix for the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + 3x_3, x_2 - x_3)$.

Is this transformation onto $\mathbb{R}^2$? (Why?)

Is this transformation one-to-one? (Why?)

6. [9 points] Each year 20% Hanover's population leaves for Norwich and the rest stay in Hanover. Also each year 40% of Norwich moves to Hanover while the rest stay in Norwich.

   (a) Write down the migration matrix.

   (b) If this year 10000 live in Hanover, 10000 in Norwich, use the matrix to compute the populations nest year.

   (c) Find a migration matrix that represents *two* year's worth of population change. [Hint: you may want to bring a factor $1/5$ out the front of the migration matrix to make this easier].