MATH 22 WORKSHEET : Eigenvectors & Eigenvalues. 8/2/06. Barnett.

A) is $\vec{x} = \begin{bmatrix} -6 \\ 5 \end{bmatrix}$ an eigenvector of $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$?

If so, what is its eigenvalue λ ?

B) is $\lambda = 3$ an eigenvalue of A ?

C) Now let $B = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$; I'll tell you the eigenvalues are $\lambda = 2, 9$

For $\boxed{\lambda = 2:}$ find a basis for the corresponding eigenspace:

what is the dimension of this eigenspace?

D) using A & $\vec{x}$ from part A), what is $A^2\vec{x}$? [Hint: don't find A^2!]

$A^k\vec{x}$?

k = any integer ≥ 1.

E) for $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$ write $(A - \lambda I)$ as a matrix, then use 2×2 det formula:

$\det(A - \lambda I) = ?$

For what values of λ does this go to zero?

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— SOLUTIONS —

A) is $\vec{x} = \begin{bmatrix} -6 \\ 5 \end{bmatrix}$ an eigenvector of $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$?

To check, multiply $A\vec{x}$ and ask if a multiple of $\vec{x}$: $A\vec{x} = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -6 \\ 5 \end{bmatrix}$

If so, what is its eigenvalue λ ? $\lambda = -4$

$= \begin{bmatrix} 24 \\ -20 \end{bmatrix} = -4\vec{x}$

B) is $\lambda = 3$ an eigenvalue of A ?

$A - 3I = \begin{bmatrix} 1-3 & 6 \\ 5 & 2-3 \end{bmatrix} = \begin{bmatrix} -2 & 6 \\ 5 & -1 \end{bmatrix} \sim \begin{bmatrix} -2 & 6 \\ 0 & 14 \end{bmatrix}$

row reduce to check if invertible ... or use $\det = ad - bc \neq 0$

2 pivots, full rank, invertible $\Rightarrow$ 3 not an eigenvalue

C) Now let $B = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$; I'll tell you the eigenvalues are $\lambda = 2, 9$

For $\boxed{\lambda = 2:}$ find a basis for the corresponding eigenspace:

$B - 2I = \begin{bmatrix} 2 & -1 & 6 \\ 2 & -1 & 6 \\ 2 & -1 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & -1/2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ so $\begin{aligned} x_1 &= \tfrac{1}{2}x_2 - 3x_3 \\ x_2 &= x_2 \\ x_3 &= x_3 \end{aligned}$

free

ie $\vec{x} = \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} x_3$

These 2 are our basis

what is the dimension of this eigenspace? $\dim \mathrm{Nul}(B - 2I) = 2$

D) using A & $\vec{x}$ from part A), what is $A^2\vec{x}$? [Hint: don't find A^2.]

$A^2\vec{x} = A(A\vec{x}) = A\lambda\vec{x} = \lambda A\vec{x} = \lambda(\lambda\vec{x})$ $A^k\vec{x}$?

$= \lambda^2\vec{x} = (-4)^2\begin{bmatrix} -6 \\ 5 \end{bmatrix} = \begin{bmatrix} -96 \\ 80 \end{bmatrix}$.

k = any integer ≥ 1.

$A^k\vec{x} = \overbrace{AA\cdots A}^{k}\vec{x} = \overbrace{A\cdots A}^{k-1}\lambda\vec{x} = \text{etc} \cdots = \lambda^k\vec{x} = (-4)^k\vec{x}$

E) for $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$ write $(A - \lambda I)$ as a matrix, then use 2x2 det formula:

$A - \lambda I = \begin{bmatrix} 2-\lambda & 3 \\ 3 & -6-\lambda \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\det(A - \lambda I) = ?$ $ad - bc = (2-\lambda)(-6-\lambda) - 3(3) = \lambda^2 + 4\lambda - 12 - 9 = \lambda^2 + 4\lambda - 21$

For what values of λ does this go to zero? $(\lambda+7)(\lambda-3) = 0$ so $\lambda = +3, -7$