HW4 Math 22

1.9 (24 points) a. False, see the paragraph preceding Example 2.

b. True, see Theorem 10.

c. True, see Table 1.

d. False, see def. of one-to-one

e. True, see the solution of Example 5.

2.1 (12 points) A suitable column for B is any multiple of (2,1), e.g., $B=\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$

2.2 (10 points) a. False, the product of inverses should be in the reverse order. c.f., Theorem 6(b).

b. True, by Theorem 6(a)

c. True, by Theorem 4.

d. True, by Theorem 7.

e. False. This is a misstatement of Theorem 7

2.2 (23 points). $Ax=0$ has only the trivial solution $\Rightarrow$ there are no free variable in $Ax=0$

Thus each column of A is a pivot column.

(33 points) $A^{-1}=B=\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \cdots & \\ \vdots & & & & \\ 0 & 0 & \cdots & -1 & 1 \end{bmatrix}$ ← This is differentiation matrix.

← note: $\begin{bmatrix} 1 & & & 0 \\ 1 & 1 & & \\ \vdots & \ddots & \ddots & \\ 1 & \cdots & 1 & 1 \end{bmatrix}$ is an integration matrix (discrete, i.e. summing) — try it.

2.3 (6 points) Not invertible. The matrix row reduces to $\begin{bmatrix} 1 & 0 & \frac{1}{3} & -4 \\ 0 & 1 & 3 & 4 \end{bmatrix}$, and is not row-equivalent to I_3

So you've proved the inverse of integration is differentiation. (discretely).

2.3 (12 points) a. True. Statement (k) of the IMT is true $\Rightarrow$ (j) is true

b True. - - - - (e) — — - - $\Rightarrow$ (b) is true.

c. True. See remark following the proof of the IMT.

d. False. If A is any $n \times n$ matrix, the linear transformation $x \mapsto Ax$ maps $\mathbb{R}^n$ to $\mathbb{R}^n$, yet not every such matrix has n pivot positions.

e. True, by IMT.

2.5 (2 points) $Ly = b \Rightarrow y = \begin{bmatrix} -2 \\ -2 \\ 2 \end{bmatrix}$, $Ux = y \Rightarrow x = \begin{bmatrix} 1/4 \\ 3 \\ 1 \end{bmatrix}$.

(10 points) $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -5 & 1 \end{bmatrix} \begin{bmatrix} -5 & 3 & 4 \\ 0 & -2 & -1 \\ 0 & 0 & 9 \end{bmatrix}$

(19 points) Row reduce $[A \; I]$

Bonus: (1 point) The inverse of an upper triangle matrix is an upper triangle matrix.

3.1 (2 points) 2.

(38 points) $\det kA = k^2 \cdot \det A$.