HW 3 — Math 22 answer key

1.8 21 (3 points) a. True. See def. before fig 2. A linear transformation is a function with certain properties.

b. False. The domain is $\mathbb{R}^5$. See the paragraph before example 1.

c. False. The range is the set of all linear combinations of the columns of A. See paragraph before example 1.

d. False. See the paragraph after the definition of a linear transformation.

e. True. See the paragraph after equation (4).

24 (3 points) 1°. Any vector $\vec{x} \in \mathbb{R}^n$ can be written as a linear combination of $v_1, \cdots, v_p$.

2° $T(x) = T(c_1v_1 + \cdots + c_pv_p) = c_1T(v_1) + \cdots + c_pT(v_p) = 0$.

30 (2 points) Let $T(x) = Ax + b$, $\forall \in \mathbb{R}^n$.

If $b \neq 0$ then $T(0) = A \cdot 0 + b \neq 0$. So T is not a linear transformation.

→ this is elegant; take note.

1.9 8 (2 points) Reflect through $x_2 = x_1$: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, reflect through x_1 axis: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

22 (3 points) $x = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

35 (2 points) Onto: $n \geq m$. One-to-one: $m \geq n$.

1.10 10 (3 points). $\begin{bmatrix} 4 \\ s_1 \end{bmatrix} = \begin{bmatrix} 754000 \\ 541000 \end{bmatrix}$, $\begin{bmatrix} c_2 \\ s_2 \end{bmatrix} = \begin{bmatrix} 722100 \\ 577900 \end{bmatrix}$.

A (2 points). In the previous year, 428571 ($\frac{3}{7}M$) people lived in the city, 571429 ($\frac{4}{7}M$) people lived in the suburb.

2.1 (16 points) a. False. $AB = [Ab_1 \ Ab_2 \ Ab_3]$.

b True. See the box after Example 6.

C False. Matrix multiplication is in general not commutative. cf P.114

d. False. In general $(AB)^T \neq A^T B^T$. cf p. 115.

e. True. See Theorem 3(b)

(17 points). First column $\begin{bmatrix} 4 \\ \end{bmatrix}$, second column $\begin{bmatrix} -8 \\ -5 \end{bmatrix}$.

Many people thought $C=D \Rightarrow m=n$.

(25 points) $I_n D = (CA)\cdot D = C\cdot(AD) = C\cdot I_m$. Thus $C=D$.

(This does not automatically imply $I_m = I_n$!).

From 23 (If $Ax=0 \Rightarrow 0 = CAx = I_n x$), $\therefore n \leq m$.

From 24 (Pf: $\forall\, b \in \mathbb{R}^m$, let $x = Db$), $\therefore m \leq n$.

Thus $m=n$.

2.2 (7 points) a. $\begin{bmatrix} -9 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 11 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 6 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 13 \\ -5 \end{bmatrix}$

b. $[A b_1 \ b_2 \ b_3 \ b_4] = \begin{bmatrix} 1 & 0 & -9 & 11 & 6 & 13 \\ 0 & 1 & 4 & -5 & -2 & -5 \end{bmatrix}$.