HW 2

1.4 4. $Ax = \begin{bmatrix} 8 & 3 & -4 \\ 5 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 1\cdot\begin{bmatrix} 8 \\ 5 \end{bmatrix} + 1\cdot\begin{bmatrix} 3 \\ 1 \end{bmatrix} + 1\cdot\begin{bmatrix} -4 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$

(2 points) $Ax = \begin{bmatrix} 8 & 3 & -4 \\ 5 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8\cdot1 + 3\cdot1 + (-4)\cdot1 \\ 5\cdot1 + 1\cdot1 + 2\cdot1 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$

10. $x_1\begin{bmatrix} 8 \\ 5 \\ 1 \end{bmatrix} + x_2\begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 8 & -1 \\ 5 & 4 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

14. NO. The equation $Ax = u$ has no solution.
(3 points)

15. The equation is not consistent when $3b_1 + b_2$ is not zero.

The set of b for which the equation is consistent is a line through the origin – the set of all points (b_1, b_2) satisfying $b_2 = -3b_1$.

(3 24 points) (a) True. See 1.4 Theorem 3

(b) True. Example 2

(c) True. Theorem 3

(d) True. See the box before example 2.

(e) False. See "warning" that follows theorem 4.

(f) True. See Theorem 4.

1.5 (3 points) 6 $x = x_3\begin{bmatrix} -4 \\ 3 \end{bmatrix}$

(3 16 points) $x = \begin{bmatrix} -5 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -4 \\ 3 \end{bmatrix} = p + x_3 q$. The solution set is the line through p, parallel to q.

(2 24 points) (a) False. A nontrivial solution of $Ax = 0$ is any nonzero x that satisfies the equation

(b) True. see example 2.

(c) True. If the zero vector is a solution, then $b = Ax = A0 = 0$

most people got this wrong. (d) True. See the paragraph following example 3.

*(e) False. Theorem 6 applies only to a consistent system.

1.6 11 (2 points) $\begin{cases} x_1 = 20 - x_3 \\ x_2 = 60 + x_3 \\ x_3 \text{ is free} \\ x_4 = 60 \end{cases}$ Since x_1 is nonnegative, the largest value of x_3 is 20.

13 12 points + bonus 2 pts) (a). $\begin{cases} x_1 = 100 + x_3 - x_5 \\ x_2 = 100 - x_3 + x_5 \\ x_3 \text{ is free} \\ x_4 = 60 - x_5 \\ x_5 \text{ is free.} \end{cases}$ (b) $\begin{cases} x_1 = 40 + x_3 \\ x_2 = 160 - x_3 \\ x_3 \text{ is free} \\ x_4 = 0 \\ x_5 = 60 \end{cases}$ (c) Minimum value of x_1 is 40 cars/minute.

1.7 12 (2 points) Linearly independent.

8. Linearly dependent since there are more variables than equations. (3 points)

10 (3 point) (a). $\begin{bmatrix} 1 & -2 & 2 \\ -5 & 10 & -9 \\ -3 & 6 & h \end{bmatrix}$ as augmented matrix is inconsistent.

Thus no h satisfies the requirement.

(b). All h satisfy the requirement.

22. (3 points) (a) True. see fig 1

(b) False. see "warning" after Theorem 8.

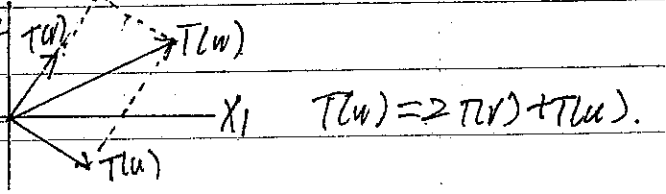
(c) True. see "remark" after Example 4.

(d). False. see example 3(a)

1.8 8. 4 columns and 5 rows.

11 (2 points) Yes, because the system represented by $[A\ b]$ is consistent.

18 (2 points) Note that $w = 2v + u$



$T(w) = 2T(v) + T(u)$.