

## Ungraded Quiz + Questionnaire 3

Your name: \_\_\_\_\_

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1. If  $V$  is a vector space and  $H \subset V$  is a subspace, can you always subtract vectors in  $H$ ? That is, if  $u, v \in H$ , does  $u - v \in H$ ? Why? **Yes.** If  $u, v \in H$ , then  $-v \in H$ . Then we have  $u + (-v) = u - v \in H$ .

2. True or false: the determinant of a matrix does not change after you perform elementary row operations. **False.** It changes after an interchange or a row scaling (if the determinant is nonzero).

3. Let  $V = \mathbb{R}^2$  and consider  $H = \{(x, y) \in V \mid xy = 0\}$ . Sketch  $H$ . Is  $H$  a subspace of  $\mathbb{R}^2$ ?

A vector  $v \in \mathbb{R}^2$  belongs to  $H$  if at least one of its coordinates is 0. If you sketch  $V$  then you get exactly the  $x$ - and  $y$ -axes. The subspace  $H$  does not equal the whole plane:  $(1, 1) \in V$  but  $(1)(1) \neq 0$ , so that  $(1, 1) \notin H$ .  $H$  is **not a subspace** of  $\mathbb{R}^2$ : while it contains the zero vector and is closed under scalar multiplication ( $(cx)(cy) = 0$  for any scalar  $c$  if  $xy = 0$ ),  $H$  is not closed under addition. That is, you can write down a pair of vectors in  $H$  whose sum is not in  $H$ . The two standard basis vectors, for example:  $(1, 0)$  and  $(0, 1)$  both lie in  $H$ , as  $(1)(0) = 0 = (0)(1)$ . But their sum is  $(1, 1)$ , which does not lie in  $H$ .