EXAM 1 SOL Sp-2014

(1)

$A = \begin{bmatrix} 3 & 6 & -9 \\ 2 & 3 & 4 \\ 0 & 2 & 5 \end{bmatrix}$ ; $\bar{b} = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix}$

$$\left[\begin{array}{cccc} 3 & 6 & -9 & 3 \\ 2 & 3 & 4 & 4 \\ 0 & 2 & 5 & -1 \end{array}\right] \xrightarrow{R1 \to R1 - R2} \left[\begin{array}{cccc} 1 & 3 & -13 & -1 \\ 2 & 3 & 4 & 4 \\ 0 & 2 & 5 & -1 \end{array}\right]$$

$$\xrightarrow{R2 \to R2 - 2R1} \left[\begin{array}{cccc} 1 & 3 & -13 & -1 \\ 0 & -3 & 30 & 6 \\ 0 & 2 & 5 & -1 \end{array}\right] \xrightarrow{R2 \to \frac{1}{3}R2} \left[\begin{array}{cccc} 1 & 3 & -13 & -1 \\ 0 & 1 & -10 & -2 \\ 0 & 2 & 5 & -1 \end{array}\right]$$

$$\xrightarrow{R3 \to R3 - 2R2} \left[\begin{array}{cccc} 1 & 3 & -13 & -1 \\ 0 & 1 & -10 & -2 \\ 0 & 0 & 25 & 3 \end{array}\right] \longrightarrow$$

$$\longrightarrow \left[\begin{array}{cccc} 1 & 3 & -13 & -1 \\ 0 & 1 & -10 & -2 \\ 0 & 0 & 1 & 3/25 \end{array}\right] \longrightarrow$$

$$\longrightarrow \left[\begin{array}{cccc} 1 & 3 & 0 & 14/25 \\ 0 & 1 & 0 & -20/25 \\ 0 & 0 & 1 & 3/25 \end{array}\right] \longrightarrow$$

$$\longrightarrow \left[\begin{array}{cccc} 1 & 0 & 0 & 74/25 \\ 0 & 1 & 0 & -4/5 \\ 0 & 0 & 1 & 3/25 \end{array}\right].$$

(a) $\bar{x} = \begin{bmatrix} 74/25 \\ -4/5 \\ 3/25 \end{bmatrix}$.

(b) $\bar{x} = \bar{0}$

(c) YES. A is invertible.

②

$$\begin{bmatrix} -2 & 2 & h-2 \\ 0 & 2 & h \\ 5 & 1 & 7 \end{bmatrix} \longrightarrow \begin{bmatrix} -2 & 2 & h-2 \\ 0 & 2 & h \\ 0 & 6 & 7 + \frac{5(h-2)}{2} \end{bmatrix}$$

ⓐ

$$\longrightarrow \begin{bmatrix} -2 & 2 & h-2 \\ 0 & 2 & h \\ 0 & 0 & 7 + \frac{5(h-2)}{2} - 3h \end{bmatrix}$$

$$7 + \frac{5h-10}{2} - 3h = 0 \iff$$

$$14 + 5h - 10 - 6h = 0 \iff$$

$$4 - h = 0 \iff h = 4.$$

ⓑ Any $h \neq 4$ makes those 3 vectors linearly independent.

(a) $7 \times 7 - 6 \times 8 = 49 - 48 = 1$

(b) $2 \times (-12 - 1) + (-1)(1 \times 1 - (-3) \times 6) =$

$= -26 - 19 = -45$

(c) $(-1)(-2) \times \det \begin{bmatrix} 0 & 4 & -11 \\ 2 & 1 & 1 \\ 0 & 5 & 0 \end{bmatrix} =$

$= 2 \times (-1)^{1+2} \times 2 \times \det \begin{bmatrix} 4 & -11 \\ 5 & 0 \end{bmatrix} = -4 \times 55$

$= -220.$

④

(a) $\frac{1}{1}\begin{bmatrix} 7 & -8 \\ -6 & 7 \end{bmatrix} = \begin{bmatrix} 7 & -8 \\ -6 & 7 \end{bmatrix}$

(b) $\left[\begin{array}{ccc|ccc} 5 & 4 & 3 & 1 & 0 & 0 \\ 4 & 3 & 5 & 0 & 1 & 0 \\ 3 & 5 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R1 \to R1 - R2}$

$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & -1 & 0 \\ 4 & 3 & 5 & 0 & 1 & 0 \\ 3 & 5 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R3 \to R3 - 3R1]{R2 \to R2 - 4R1}$

$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & -1 & 0 \\ 0 & -1 & 13 & -4 & 5 & 0 \\ 0 & 2 & 10 & -3 & 3 & 1 \end{array}\right] \xrightarrow{R3 \to R3 + 2R2}$

$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & -1 & 0 \\ 0 & -1 & 13 & -4 & 5 & 0 \\ 0 & 0 & 36 & -11 & 13 & 1 \end{array}\right] \xrightarrow[R2 \to -R2]{R3 \to \frac{1}{36} R3}$

$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & -1 & 0 \\ 0 & 1 & -13 & 4 & -5 & 0 \\ 0 & 0 & 1 & -11/36 & 13/36 & 1/36 \end{array}\right]$

④ part 2

$$\longrightarrow \begin{bmatrix} 1 & 1 & 0 & 7/18 & -5/18 & 1/18 \\ 0 & 1 & 0 & 1/36 & -11/36 & 13/36 \\ 0 & 0 & 1 & -11/36 & 13/36 & 1/36 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 0 & 13/36 & +11/36 & -11/36 \\ 0 & 1 & 0 & 1/36 & -11/36 & 13/36 \\ 0 & 0 & 1 & -11/36 & 13/36 & 1/36 \end{bmatrix}$$

$$A^{-1} = \frac{1}{36}\begin{bmatrix} 13 & 1 & -11 \\ 1 & -11 & 13 \\ -11 & 13 & 1 \end{bmatrix}$$

ⓒ

$$\overline{x} = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 13 \end{bmatrix}.$$

⑤

⑤ $A = \begin{bmatrix} 7 & 8 \\ 6 & 7 \end{bmatrix}$

ⓐ $A \longrightarrow \begin{bmatrix} 7 & 8 \\ 0 & 1/7 \end{bmatrix} = U$

$L = \begin{bmatrix} 1 & 0 \\ 6/7 & 1 \end{bmatrix}.$

ⓑ $U = \begin{bmatrix} 7 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1/7 \end{bmatrix} \begin{bmatrix} 1 & 8/7 \\ 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 \\ 6/7 & 1 \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1/7 \end{bmatrix} \begin{bmatrix} 1 & 8/7 \\ 0 & 1 \end{bmatrix}.$

ⓒ $A^{-1} = \begin{bmatrix} 1 & -8/7 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} 1/7 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -6/7 & 1 \end{bmatrix}$

(6)

(6) (a) T

(b) $\det \begin{vmatrix} 1 & 0 & 2 & 3 & -1 \\ 1 & 0 & 2 & 8 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 2 & 4 & 4 \\ 2 & 0 & 4 & 5 & 11 \end{vmatrix} =$

$= \det \begin{vmatrix} 1 & 0 & 2 & 3 & -1 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 2 & 4 & 4 \\ 2 & 0 & 4 & 5 & 11 \end{vmatrix} =$

$= (-1)^{2+4} \cdot 5 \begin{vmatrix} 1 & 0 & 2 & -1 \\ 0 & 0 & 0 & 2 \\ 1 & 1 & 2 & 4 \\ 2 & 0 & 4 & 11 \end{vmatrix} = 5(-1)^{2+4} \cdot 2 \begin{vmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 2 & 0 & 4 \end{vmatrix} =$

$= 10 \begin{vmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 0 & 0 & 0 \end{vmatrix} = 0.$

(b) F

(c) T

(d) F

7

(a) $T: \mathbb{R}^2 \to \mathbb{R}^3$; $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ 2x_1 - 5x_2 \\ x_1 + x_2 \end{bmatrix}$

$$T(\bar{x}) = x_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ -5 \\ 1 \end{bmatrix}.$$

So associated matrix is $A = \begin{bmatrix} 1 & -2 \\ 2 & -5 \\ 1 & 1 \end{bmatrix}$.

(a) Domain $\mathbb{R}^2$

(b) Codomain $\mathbb{R}^3$

(c) $A = \begin{bmatrix} 1 & -2 \\ 2 & -5 \\ 1 & 1 \end{bmatrix}$

(d) A has one pivot in every column:

$$A \longrightarrow \begin{bmatrix} 1 & -2 \\ 0 & -1 \\ 0 & 3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

So, T is not onto (not a pivot in every row)

(e) T is one-to-one. A has a pivot in every column.

(8)

(9) $T(\bar{e}_1) = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$ ; $T(\bar{e}_2) = \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix}$ ; $T(\bar{e}_3) = \begin{bmatrix} 6 \\ 7 \\ -1 \end{bmatrix}$

$$A = \begin{bmatrix} 4 & 5 & 6 \\ 3 & -1 & 7 \\ 1 & 0 & -1 \end{bmatrix}$$

(a) $T\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 3 & -1 & 7 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 32 \\ 22 \\ -2 \end{bmatrix}$

(b) ? $\det A = 1 \cdot \begin{vmatrix} 5 & 6 \\ -1 & 7 \end{vmatrix} + (-1) \begin{vmatrix} 4 & 5 \\ 3 & -1 \end{vmatrix} = (35+6) - (-4-15)$
$= 41 + 19 = 60 \neq 0$

YES, T is invertible.

(g) $A(B+C) = A(B+D)$.

A is invertible so multiplying the above by $A^{-1}$ we have

$A^{-1}(A(B+C)) = A^{-1}(A(B+D))$ or

$(A^{-1}A)(B+C) = (A^{-1}A)(B+D)$ or

$B+C = B+D$ so

$\boxed{C = D}$