Homework Assignment for §3.1–3.3

Math 22, Spring 2007

1. Show $\det(E) = k$, where E is the $n \times n$ elementary matrix representing multiplication of row ℓ by k. That is, $E = [e_{ij}]$ is the $n \times n$ diagonal matrix with $e_{\ell\ell} = k$ and all other diagonal entries 1.

2. Use cofactor expansion (*not* row reduction) to show that $\det(E) = -1$, where E is the $n \times n$ elementary matrix representing the interchange of rows k and ℓ. That is, for $E = [e_{ij}]$, $e_{k\ell} = e_{\ell k} = 1$, $e_{ii} = 1$ for $i \neq k, \ell$, and all other entries are 0.

   Hint: Pare E down via cofactor expansion until the submatrix under consideration is
   $$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

3. Use cofactor expansion to show $\det(E) = 1$, where E is the $n \times n$ elementary matrix representing addition of b times row ℓ to row k. That is, $E = [e_{ij}]$ where $e_{ii} = 1$ for all i, $e_{k\ell} = b$, and all other entries are 0.

   Hint: Use the first round of cofactor expansion to eliminate b entirely. What does your submatrix look like?

4. Explain the connection between the results you found in 1–3 and the effect of row operations on the value of $\det(A)$.

5. Read the second half of §3.3: "Determinants as Area and Volume" and "Linear Transformations".

From the book:

§3.1 p. 190 #13
§3.2 p. 199 #15–20, 29, 31 (15–20 should take no time at all).
§3.3 p. 209 #19, 27, 29