NAME: ______________________

MATH 20 QUIZ 2

May 15, 2014

INSTRUCTIONS: This is a closed book, closed notes, computer-free quiz. You are not to give nor to receive help from any outside source during the exam. Remember that your instructors can clarify any questions that are not clear to you.

Please show all of your work and justify all of your answers.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own work.

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Signature

Question	Points	Score
1	8	
2	14	
3	8	
4	10	
5	0	
Total:	40	

1. Let X be a non-negative random variable with $\mathbb{E}(X) = 5$.

   (a) [4 points] Given this information, what can you say about

$$\mathbb{P}(X \geq 8)?$$

Markov ineq.

$$\mathbb{P}(X \geq \underset{8}{a}) \leq \frac{\underset{5}{\mathbb{E}(X)}}{a}$$

$$= \frac{5}{8}$$

   (b) [4 points] If you know that X is an exponential random variable, what is

$$\mathbb{P}(X \geq 8)? \qquad \lambda = \frac{1}{5}$$

$$\mathbb{P}(X \geq 8) = 1 - \mathbb{P}(X \leq 8)$$

$$= 1 - (1 - e^{-8/5}) \quad \text{CDF}$$

$$= e^{-8/5}$$

2. Let S_n be the number of successes on n Bernoulli trials with probability .6 of success on each trial. Let $A_n = S_n/n$ be the average number of successes. Determine the following and justify your answer:

(a) [4 points] $\lim_{n\to\infty} P(A_n = .6)$ $= 0$

$$\mathbb{P}(A_n = .6) = b(n, .6, .6n) \to 0$$

(b) [4 points] $\lim_{n\to\infty} P(.5n < S_n < .7n)$

$$\mathbb{P}(.5n < S_n < .7n)$$

$$= \mathbb{P}\left(\left| \frac{S_n}{n} - .6 \right| \geq .1 \right) \to 0$$

LoLN

(c) [6 points] $\lim_{n\to\infty} P(S_n < .6n + .6\sqrt{(n)})$

$$\mathbb{P}(S_n < .6n + .6\sqrt{n})$$

$$= \mathbb{P}\left(\frac{S_n - .6n}{\sqrt{n(.6)(.4)}} < \frac{.6\sqrt{n}}{\sqrt{n(.6)(.4)}} \right)$$

$$\to \mathbb{P}\left(Z < \frac{.6}{\sqrt{.24}} \right) = \frac{1}{2} + NA\left(\frac{.6}{\sqrt{.24}} \right)$$

std. normal

3. [8 points] Let X and Y be independent uniformly distributed random variables on the unit interval and $U = \max(X, Y)$. Find the cumulative distribution function of U.

CDF

$$F_U(a) = \mathbb{P}(U \leq a)$$
$$= \mathbb{P}(X \leq a \cap Y \leq a)$$
$$= \mathbb{P}(X \leq a)\,\mathbb{P}(Y \leq a)$$
$$= F_X(a)\,F_Y(a)$$
$$= \begin{cases} 0 & a < 0 \\ a^2 & 0 < a < 1 \\ 1 & a > 1 \end{cases}$$

4. A bank accepts rolls of pennies and gives 50 cents credit to a customer without counting the contents. Assume a roll contains 49 pennies 30% of the time, 50 pennies 60% of the time and 51 pennies the remaining 10%.

   (a) [4 points] Find the expected value and the variance of the amount the bank loses on a typical roll.

X = loss

lose 1 gain 1

$$\mathbb{E}(X) = 1(.3) - 1(.1) = .2$$

$$V(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$$

$$= 1(.3) + 1(.1) - (.2)^2$$

$$= .4 - .04 = .36$$

   (b) [6 points] Estimate the probability the bank loses 16 cents or less on 100 rolls.

$$\mathbb{P}(S_{100} < 16) = \mathbb{P}\left(\frac{S_{100} - 20}{\sqrt{100(.36)}} < \frac{16-20}{\sqrt{100(.36)}} \right)$$

$$\overset{\text{CLT}}{\approx} \mathbb{P}\left(Z < \frac{-4}{6}\right) = \frac{1}{2} - NA\left(\frac{2}{3}\right)$$

std normal

$$\approx \frac{1}{2} - .2486$$

$$= .2514$$