1)

X \ Y	1	2	3	4	5	6
1	$1/6$					
2	$1/12$	$1/12$				
3	$1/18$	$1/18$	$1/18$			
4	$1/24$	$1/24$	$1/24$	$\frac{1}{24}$		
5	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	
6	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
	$\frac{147}{360}$	$\frac{87}{360}$	$\frac{57}{360}$	$\frac{37}{360}$	$\frac{22}{360}$	$\frac{10}{360}$

joint distribution

marginal distribution of Y (late addition)

2) a) Let $X = \#$ rolls.

$$\mathbb{P}(X \geq 4 \mid X > 1) = \frac{\mathbb{P}(X \geq 4 \cap X \geq 2)}{\mathbb{P}(X \geq 2)} = \frac{\mathbb{P}(X \geq 4)}{\mathbb{P}(X \geq 2)}$$

$$= \frac{(5/6)^3}{5/6} = 25/36$$

Alternatively, given first roll is not 6, need 2nd & 3rd rolls to not be 6. Then

$$\mathbb{P}(X \geq 4) = \mathbb{P}(\text{2nd, 3rd rolls aren't } 6) \underset{\text{independence}}{=} \mathbb{P}(\text{2nd roll not } 6)\,\mathbb{P}(\text{3rd roll} \neq 6)$$

$$= \frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36}$$

2) b) $\mathbb{P}(\text{I win}) = \mathbb{P}(\text{first 6 on odd roll})$

$$= \sum_{i=0}^{\infty} \mathbb{P}(X = 2i+1)$$

$$= \sum_{i=0}^{\infty} \left(\tfrac{5}{6}\right)^{2i} (1/6)$$

$$= \frac{1}{6} \sum_{i=0}^{\infty} \left(\frac{25}{36}\right)^{i} = \frac{1}{6} \cdot \frac{1}{1 - 25/36} = \frac{1}{6} \cdot \frac{36}{11} = 6/11$$

Alternatively,

$$\mathbb{P}(\text{I win}) = \mathbb{P}(\text{I win on roll one}) + \mathbb{P}(\text{I win going second})(1 - \tfrac{1}{6})$$

$$= \mathbb{P}(\text{I win on roll one}) + \mathbb{P}(\text{you win}) \cdot 5/6$$

$$= \mathbb{P}(\text{I win on roll one}) + (1 - \mathbb{P}(\text{I win}))\, 5/6$$

so $\frac{11}{6} \mathbb{P}(\text{I win}) = 1/6 + 5/6$ so $\mathbb{P}(\text{I win}) = 6/11$

c) $\mathbb{P}(\overset{B}{\text{you win}}) = 1 - \mathbb{P}(\text{I win}) = 5/11$

$\mathbb{P}(\overset{A}{\text{you win on your first roll}}) = \frac{5}{6} \cdot \frac{1}{6} = 5/36$

Then $\mathbb{P}(A|B) = \dfrac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \dfrac{5/36}{5/11} = \boxed{11/36}$

3) $\Omega = \{d_1, d_2, d_3\}$ w/ uniform measure

$$\mathbb{P}(d_i \mid +) = \frac{\mathbb{P}(+ \mid d_i)\,\mathbb{P}(d_i)}{\sum_{i=1}^{3} \mathbb{P}(+ \mid d_i)\,\mathbb{P}(d_i)} \Bigg\} \text{ Bayes Thm}$$

$$= \frac{\mathbb{P}(+ \mid d_i) \cdot 1/3}{\sum_{i=1}^{3} \mathbb{P}(+ \mid d_i) \cdot 1/3} = \frac{\mathbb{P}(+ \mid d_i)}{.8 + .6 + .4}$$

For $i=1$, $\mathbb{P}(d_1 \mid +) = \frac{.8}{1.8} = 4/9$

$i=2$ $\mathbb{P}(d_2 \mid +) = \frac{.6}{1.8} = 3/9 = 1/3$

$i=3$ $\mathbb{P}(d_3 \mid +) = \frac{.4}{1.8} = 2/9$

4) Let $J = \{$John's hand is better$\}$, $M = \{$Mary's hand is better$\}$

$R = \{$Mary raises$\}$

Given John's assumptions,

$$\mathbb{P}(M \mid R) = \frac{\mathbb{P}(R \mid M)\,\mathbb{P}(M)}{\mathbb{P}(R \mid M)\,\mathbb{P}(M) + \mathbb{P}(R \mid J)\,\mathbb{P}(J)}$$

$$= \frac{(.9)(.04)}{.9(.04) + .1(.96)} = \frac{.036}{.036 + .096} = \frac{27}{99}$$

5) Let $N = \{\text{ordinary coin chosen}\}$, $D = \{\text{2 heads coin chosen}\}$

$$\mathbb{P}(D \mid H^6) = \frac{\mathbb{P}(H^6 \mid D)\,\mathbb{P}(D)}{\mathbb{P}(H^6 \mid D)\,\mathbb{P}(D) + \mathbb{P}(H^6 \mid N)\,\mathbb{P}(N)} = \frac{1 \cdot 1/65}{\frac{1}{65} + \left(\frac{1}{2}\right)^6 \cdot \frac{64}{65}} = \frac{\frac{1}{65}}{\frac{1}{65} + \frac{1}{65}} = 1/2.$$

6) For all j, $\mathbb{P}(X_j = 1) = \dfrac{6 \leftarrow 3! \text{ permutations w/ } j\text{th position} = j}{24 \leftarrow 4! \text{ permutations}}$

$$\mathbb{P}(X_j = 0) = 3/4$$

$$\mathbb{P}(X_i = 1 \mid X_j = 1) = \frac{\mathbb{P}(X_i = 1, X_j = 1)}{\mathbb{P}(X_j = 1)} = \frac{2/24}{1/4} = 1/3 \neq \mathbb{P}(X_i = 1)$$

permutations w/ ith pos $= i$, jth pos $= j$

so <u>not</u> independent.

7) a) By symmetry, can assume we pick door A

Monte reveals door B

$$\Omega = \{ A_C B_{MG} C_G,\ A_G B_{MC} C_G,\ A_G B_{MG} C_G \}$$

car in A (A_C); Monte reveals goat (B_{MG})

$\mathbb{P}(A_C \mid B_{MG}) = \dfrac{1}{2} = \mathbb{P}(A_G \mid B_{MG})$, so switching has no effect now!

7) b) We choose A.

$$\Omega = \{ A_c B_{MG} C_G,\ A_c B_G C_{MG},\ A_G B_c C_{MG},\ A_G B_{MG} C_c \}$$

$$m(A_G B_c C_{MG}) = .4, \quad m(A_G B_{MG} C_c) = .15$$

$$\underset{\overset{\shortparallel}{.225}}{m(A_c B_{MG} C_G)} + \underset{\overset{\shortparallel}{.225}}{m(A_c B_G C_{MG})} = .45$$

Assume (some assumption must be made)

Then $\mathbb{P}(A_G \mid B_{MG}) = \dfrac{.15}{.15 + .225} = \dfrac{6}{15} = \dfrac{2}{5}$ don't switch

win if switch

$\mathbb{P}(A_G \mid C_{MG}) = \dfrac{.4}{.4 + .225} = \dfrac{16}{25}$ switch

$$\mathbb{P}(\text{win choosing } A) = \frac{3}{5} \cdot \mathbb{P}(B_{MG}) + \frac{16}{25} \cdot \mathbb{P}(C_{MG})$$

Similarly, choosing B get

$$\Omega = \{ \overset{.45}{A_c B_G C_{MG}},\ \overset{.2}{A_{MG} B_c C_G},\ \overset{.2}{A_G B_c C_{MG}},\ \overset{.15}{A_{MG} B_G C_c} \}$$

$\mathbb{P}(B_G \mid A_{MG}) = \dfrac{.15}{.15 + .2} = \dfrac{3}{7}$ don't switch

$\mathbb{P}(B_G \mid C_{MG}) = \dfrac{.45}{.45 + .2} = \dfrac{9}{13}$ switch $\mathbb{P}(\text{win choosing } B)$

$= \dfrac{4}{7} \mathbb{P}(A_{MG}) + \dfrac{9}{13} \mathbb{P}(C_{MG})$

Choosing C, get

$$\Omega = \{ \overset{.45}{A_c B_{MG} C_G},\ \overset{.4}{A_{MG} B_c C_G},\ \overset{.075}{A_{MG} B_G C_c},\ \overset{.075}{A_G B_{MG} C_c} \}$$

$\mathbb{P}(C_G \mid A_{MG}) = \dfrac{.45}{.45 + .075} = \dfrac{18}{21}$, $\mathbb{P}(C_G \mid B_{MG}) = \dfrac{.4}{.4 + .075} = \dfrac{16}{19}$ switch both

$\mathbb{P}(\text{win choosing } C) = \dfrac{18}{21} \cdot \mathbb{P}(A_{MG}) + \dfrac{16}{19} \mathbb{P}(B_{MG})$ ← best!