

Homework 2

Due April 9, 2014

Please make sure to explain your answers to each of the following questions. Remember: a correct numerical answer without explanation is worth no points! Write up your answers legibly and logically. The not-to-turn-in problems provide additional practice and are important to preparing for exams.

1. A car is parked among N cars in a row, not on either end. Upon returning, the owner finds that exactly r of the N places are still occupied. What is the probability that both places neighboring her car are empty?
2. In Bridge, four players are each dealt a hand of thirteen cards.
 - (a) A hand is called a *Yarborough* if it has no aces, kings, queens, jacks or tens. What is the probability that a given hand is a Yarborough?
 - (b) How many ways are there to deal out the four bridge hands?
3. Section 3.2, Exercise 31
4. `coinTosses`: A single trial constitutes flipping `run_time` fair coins. It is a success if the proportion of heads is between `.4` and `.6`. Use `coinTosses` or `rbinom` to answer the following:
 - (a) Estimate the probability of success for `run_time = 11, 101, 1001`.
 - (b) Let p be your estimate for the probability that after 101 flips, the proportion of heads is between `.4` and `.6`. Find values a, b so that the probability that, after 1001 flips, the proportion of heads is between a and b is approximately p .
5. The Polya Urn model begins with one red ball and one blue ball in an urn. At each round, a ball is drawn, then returned with another of the same color. Let X_n be the number of red balls after n draws. Show that X_n is a uniformly random number between 1 and n . (Hint: First find the probability of drawing k red balls and $n - k$ blue balls. Then show the order the balls are drawn in does not affect this probability.)

Problems **not** to turn in (Items with * go beyond practice):

1. Section 3.2 Exercise 10
2. Section 3.2 Exercise 22
3. The game of Poker Dice is played by simultaneously rolling 5 dice. Show (rounding to 4 digits):
 - (a) $P(\text{no two alike}) = .0926$
 - (b) $P(\text{one pair}) = .4630$
 - (c) $P(\text{two pair}) = .2315$
 - (d) $P(\text{three alike}) = .1543$
 - (e) $P(\text{full house}) = .0386$
 - (f) $P(\text{four alike}) = .0193$
 - (g) $P(\text{five alike}) = .0008$
4. * Smoking problems. A smoker keeps two matchbooks containing n matches, one in each pocket. Each time he lights up, he reaches into a pocket at random and uses a match.
 - (a) What is the probability, upon finding the first matchbook that he looks at to be empty, that the second matchbook is empty as well?
 - (b) Extending the above, find a distribution for the number of matches left in when the first matchbook is empty.
 - (c) Try simulating to see, for $n = 100$, the probability this number is less than 10.