Here are some questions about independence.

1. Toss two 6-sided dice. Let X_1 be the value of the first and X_2 be the value of the second. Are the events A and B independent for:

   (a) $A = \{X_1 = 4\}$ and $B = \{X_2 = 3\}$

   $P(A) = 1/6$ $\quad P(A \cap B) = \frac{1}{36}$ yes!
   $P(B) = 1/6$

   (b) $A = \{X_1 = 4\}$ and $B = \{X_1 + X_2 = 6\}$ $\quad$ 6 $X_1=4, X_2=2$ no!

   $P(A) = 1/6$
   $P(B) = 5/36$ $\quad P(A \cap B) = \frac{1}{36}$

   (c) $A = \{X_1 = 4\}$ and $B = \{X_1 + X_2 = 7\}$

   $P(A) = 1/6$
   $P(B) = 1/6$ $\quad P(A \cap B) = 1/36$ yes!

2. A family has k kids. Let Y_1 be the number of girls and Y_2 be the number of genders represented (we assume a gender binary, sorry). Let $A = \{Y_1 \geq m\}$ and $B = \{Y_2 = 2\}$. Are A and B independent when:

   (a) $k = 3$ and $m = 1$

   $P(A) = \frac{7}{8}$ $\quad P(A \cap B) = \frac{6}{8}$ no.
   $P(B) = \frac{6}{8}$

   (b) $k = 4$ and $m = 2$

   $P(A) = \frac{11}{16}$ $\leftarrow \binom{4}{2} + \binom{4}{3} + \binom{4}{4}$ $\quad P(A \cap B) = \frac{10}{16}$ no
   $P(B) = 14/16$

   (c) $k = 2$ and $m = 3$

   $P(A \cap B) = 0$, so depends on definition

   (need more girls than kids)